



arXiv:2012.07905

Freie Universität Berlin

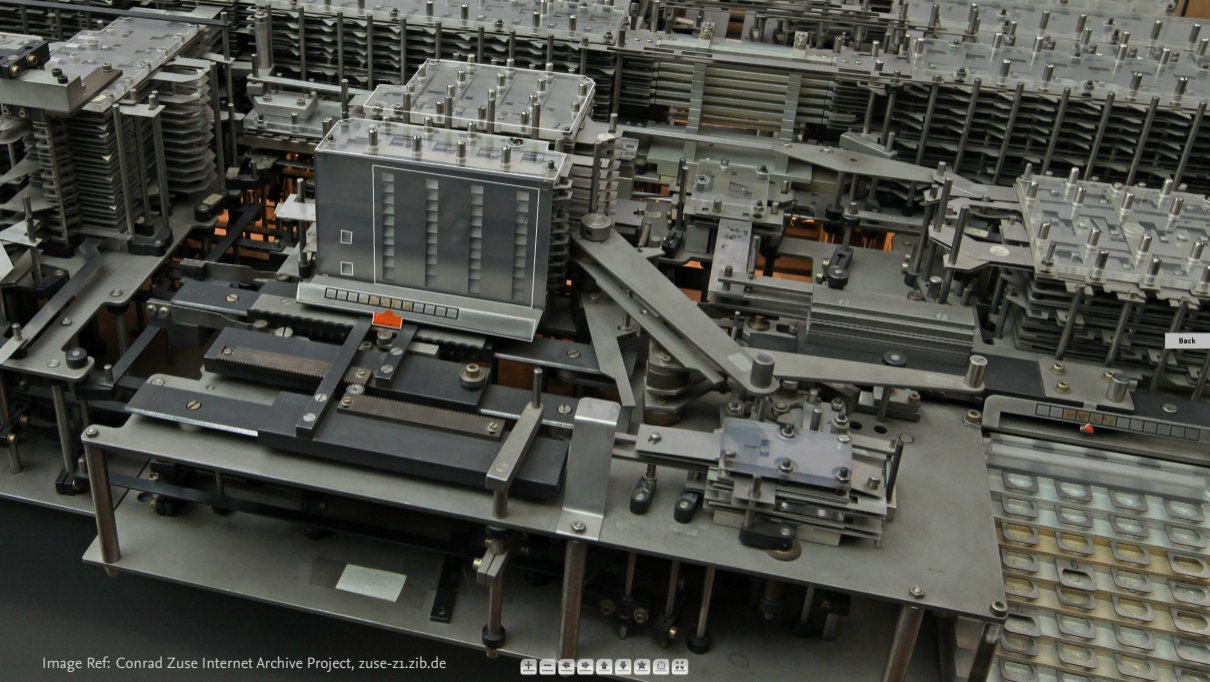


The quantum sign problem

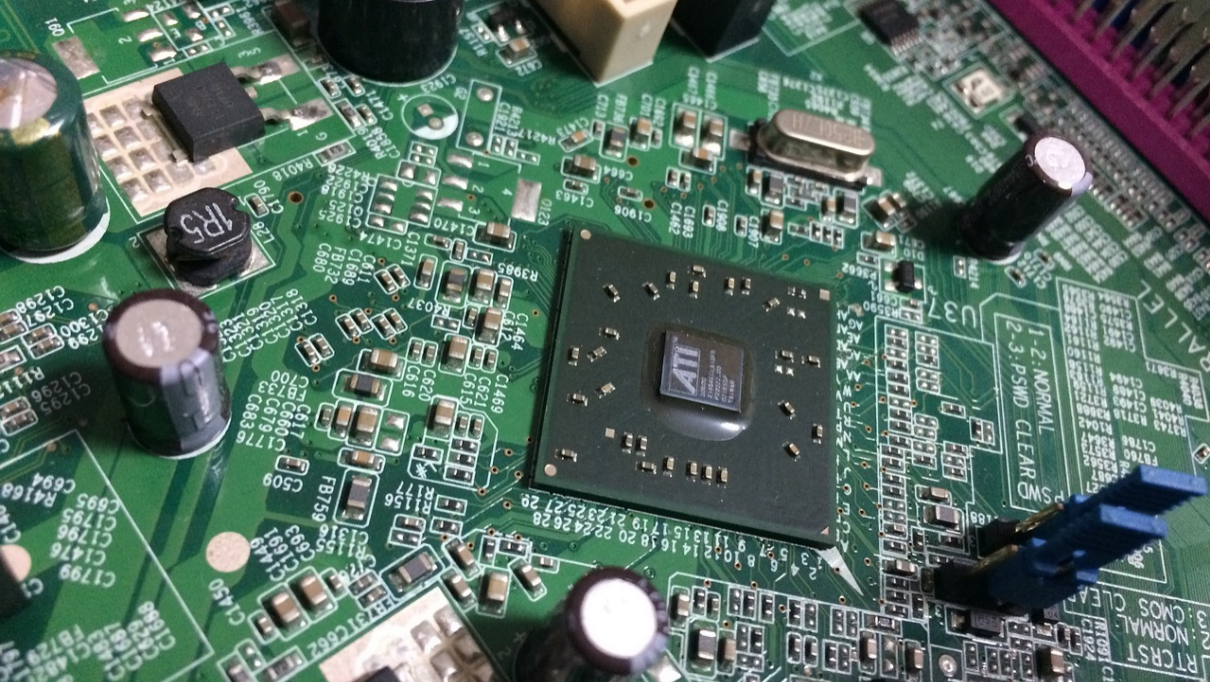
Dominik Hangleiter

IQC-QuICS Seminar, February 16, 2021

Licensed under CC BY-NC 4.0.



Back



MT
0513 02024
02018

1-2: NORMAL
2-3: PSWD CLEAR

RTCRST
2: NORMAL
3: CMOS CLEAR

U37
PSWD
188

AGAN
M
W
R
Z
C
E
M
C
A

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

C1908 C1482 C1483 C1484 C1485 C1486 C1487 C1488 C1489 C1490 C1491 C1492 C1493 C1494 C1495 C1496 C1497 C1498 C1499 C1500 C1501 C1502 C1503 C1504 C1505 C1506 C1507 C1508 C1509 C1510 C1511 C1512 C1513 C1514 C1515 C1516 C1517 C1518 C1519 C1520 C1521 C1522 C1523 C1524 C1525 C1526 C1527 C1528 C1529 C1530 C1531 C1532 C1533 C1534 C1535 C1536 C1537 C1538 C1539 C1540 C1541 C1542 C1543 C1544 C1545 C1546 C1547 C1548 C1549 C1550 C1551 C1552 C1553 C1554 C1555 C1556 C1557 C1558 C1559 C1560 C1561 C1562 C1563 C1564 C1565 C1566 C1567 C1568 C1569 C1570 C1571 C1572 C1573 C1574 C1575 C1576 C1577 C1578 C1579 C1580 C1581 C1582 C1583 C1584 C1585 C1586 C1587 C1588 C1589 C1590 C1591 C1592 C1593 C1594 C1595 C1596 C1597 C1598 C1599 C1600 C1601 C1602 C1603 C1604 C1605 C1606 C1607 C1608 C1609 C1610 C1611 C1612 C1613 C1614 C1615 C1616 C1617 C1618 C1619 C1620 C1621 C1622 C1623 C1624 C1625 C1626 C1627 C1628 C1629 C1630 C1631 C1632 C1633 C1634 C1635 C1636 C1637 C1638 C1639 C1640 C1641 C1642 C1643 C1644 C1645 C1646 C1647 C1648 C1649 C1650 C1651 C1652 C1653 C1654 C1655 C1656 C1657 C1658 C1659 C1660 C1661 C1662 C1663 C1664 C1665 C1666 C1667 C1668 C1669 C1670 C1671 C1672 C1673 C1674 C1675 C1676 C1677 C1678 C1679 C1680 C1681 C1682 C1683 C1684 C1685 C1686 C1687 C1688 C1689 C1690 C1691 C1692 C1693 C1694 C1695 C1696 C1697 C1698 C1699 C1700 C1701 C1702 C1703 C1704 C1705 C1706 C1707 C1708 C1709 C1710 C1711 C1712 C1713 C1714 C1715 C1716 C1717 C1718 C1719 C1720 C1721 C1722 C1723 C1724 C1725 C1726 C1727 C1728 C1729 C1730 C1731 C1732 C1733 C1734 C1735 C1736 C1737 C1738 C1739 C1740 C1741 C1742 C1743 C1744 C1745 C1746 C1747 C1748 C1749 C1750 C1751 C1752 C1753 C1754 C1755 C1756 C1757 C1758 C1759 C1760 C1761 C1762 C1763 C1764 C1765 C1766 C1767 C1768 C1769 C1770 C1771 C1772 C1773 C1774 C1775 C1776 C1777 C1778 C1779 C1780 C1781 C1782 C1783 C1784 C1785 C1786 C1787 C1788 C1789 C1790 C1791 C1792 C1793 C1794 C1795 C1796 C1797 C1798 C1799 C1800 C1801 C1802 C1803 C1804 C1805 C1806 C1807 C1808 C1809 C1810 C1811 C1812 C1813 C1814 C1815 C1816 C1817 C1818 C1819 C1820 C1821 C1822 C1823 C1824 C1825 C1826 C1827 C1828 C1829 C1830 C1831 C1832 C1833 C1834 C1835 C1836 C1837 C1838 C1839 C1840 C1841 C1842 C1843 C1844 C1845 C1846 C1847 C1848 C1849 C1850 C1851 C1852 C1853 C1854 C1855 C1856 C1857 C1858 C1859 C1860 C1861 C1862 C1863 C1864 C1865 C1866 C1867 C1868 C1869 C1870 C1871 C1872 C1873 C1874 C1875 C1876 C1877 C1878 C1879 C1880 C1881 C1882 C1883 C1884 C1885 C1886 C1887 C1888 C1889 C1890 C1891 C1892 C1893 C1894 C1895 C1896 C1897 C1898 C1899 C1900 C1901 C1902 C1903 C1904 C1905 C1906 C1907 C1908 C1909 C1910 C1911 C1912 C1913 C1914 C1915 C1916 C1917 C1918 C1919 C1920 C1921 C1922 C1923 C1924 C1925 C1926 C1927 C1928 C1929 C1930 C1931 C1932 C1933 C1934 C1935 C1936 C1937 C1938 C1939 C1940 C1941 C1942 C1943 C1944 C1945 C1946 C1947 C1948 C1949 C1950 C1951 C1952 C1953 C1954 C1955 C1956 C1957 C1958 C1959 C1960 C1961 C1962 C1963 C1964 C1965 C1966 C1967 C1968 C1969 C1970 C1971 C1972 C1973 C1974 C1975 C1976 C1977 C1978 C1979 C1980 C1981 C1982 C1983 C1984 C1985 C1986 C1987 C1988 C1989 C1990 C1991 C1992 C1993 C1994 C1995 C1996 C1997 C1998 C1999 C2000

circuitOptimizer.py

```

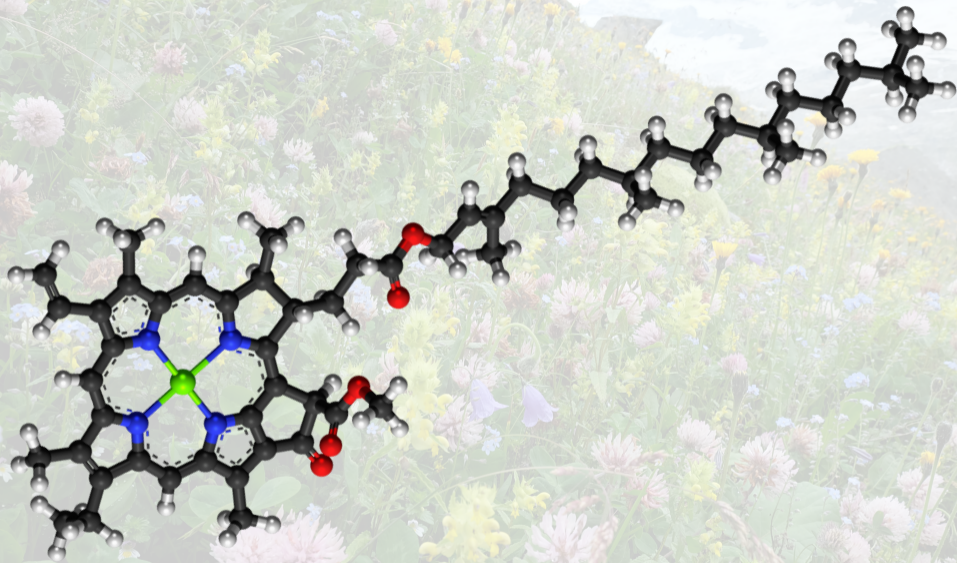
917 orth = circuit.coreOrths[k]
918 newCircuit = copy.deepcopy(circuit)
919
920 P = polynom_order
921 # get period and steps
922 o = np.max(np.abs(np.linalg.eigvalsh(stepDir)))
923 t = 2*np.pi/fct_order/o
924 mu = np.arange(P+1)*t/P
925 # get translations
926 r_s = sla.expm(-mu[1]*stepDir)
927 R = [np.identity(orth.shape[0], dtype=np.complex128)]
928 for i in range(1,P+1):
929     R.append(np.dot(R[-1],r_s))
930 # get derivatives
931 def _derivative(R):
932     newCircuit.coreOrths[k] = R.dot(orth)
933     return -2*np.real(np.trace(self.calculateEuclideanGradient(newCircuit, k).dot(orth.T.conj()).dot(R.T.conj()).dot(stepDir.T.conj())))
934 dervs = list(map(_derivative,R))
935 mu_mat_inv = np.linalg.inv(mu[1:np.newaxis]**np.arange(1,P+1))
936 coef = mu_mat_inv.dot(dervs[1:] - dervs[0])
937 coef = list(coef[:,~1])
938 coef.append(dervs[0])
939 roots = np.roots(coef)
940 pos_real_roots = [np.real(r) for r in roots if np.abs(np.imag(r))<1E-10 and np.real(r)>=0]
941
942 if showPlot is True:
943     gridSize = 200
944     normPortion = .5
945     orth0 = circuit.coreOrths[k]
946     newCircuit = copy.deepcopy(circuit)
947     gradient = stepDir
948
949     riemGradNormSqr = .5*np.trace(np.dot(gradient.T,gradient))
950     stepSize = normPortion/ riemGradNormSqr
951
952     xgrid = np.arange(0,gridSize)*stepSize
953     objval = np.zeros((gridSize))
954
955     for kk in range(0,len(xgrid)):
956         theOrth = np.dot(sla.expm(-xgrid[kk]*gradient), orth0)
957         newCircuit.coreOrths[0] = theOrth
958         objval[kk] = self.objectiveFunction(newCircuit)

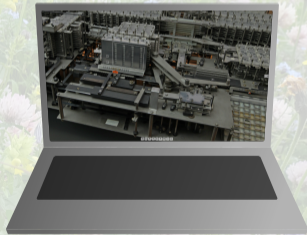
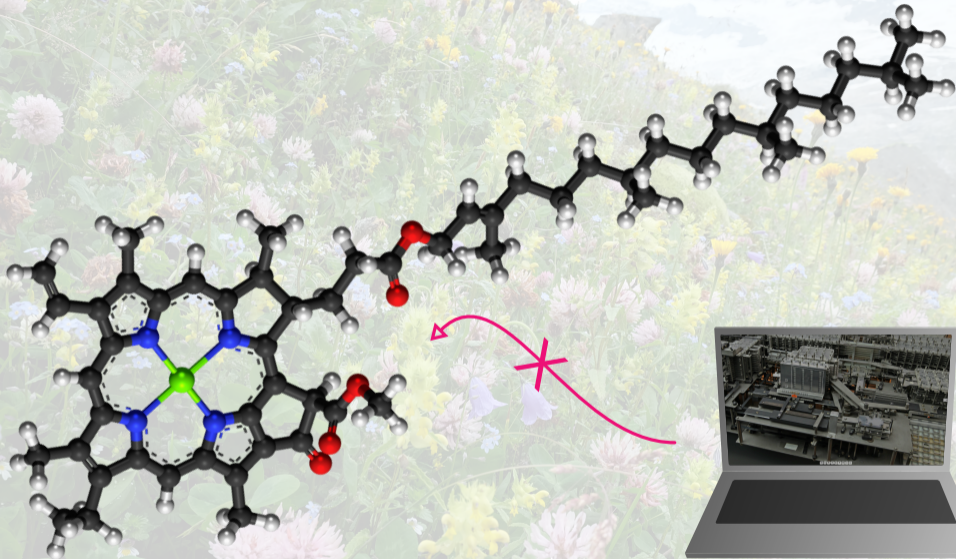
```

NORMAL >> circuitOptimizer.py

python utf-8[unix] < 84% : 958: 5 < ! trailing[4]

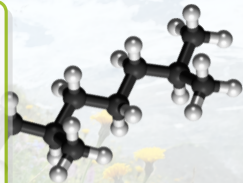
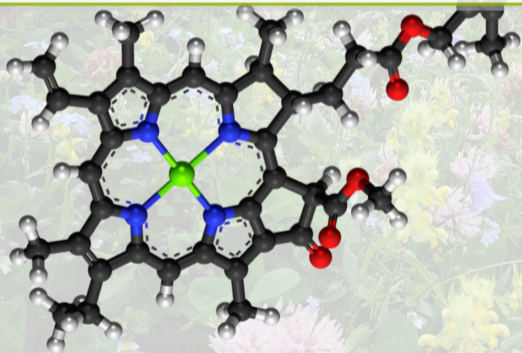






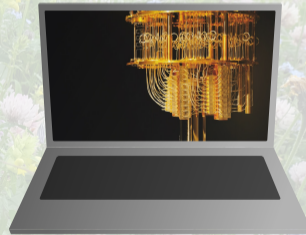
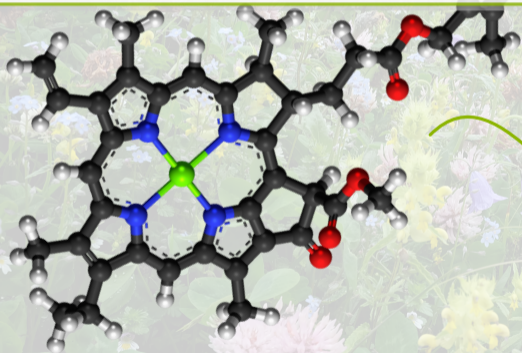
“We can give up on our rule about what the computer was, we can say: Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.”

— Richard Feynman



“We can give up on our rule about what the computer was, we can say: Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.”

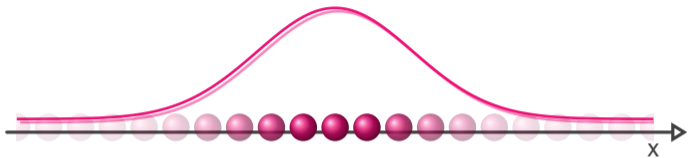
— Richard Feynman



Quantum measurement

Complex-valued wave function:

$$\psi : \mathbb{R} \rightarrow \mathbb{C}$$



Quantum measurement

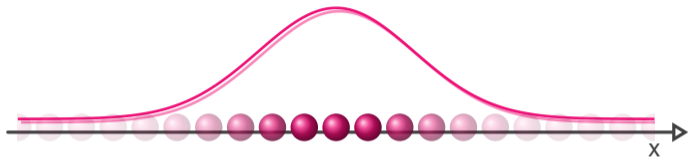
Complex-valued wave function:

$$\psi : \mathbb{R} \rightarrow \mathbb{C}$$



Wave-function collapse:

$$\Pr[\text{measured at } x] = |\psi(x)|^2$$



Quantum measurement

Complex-valued wave function:

$$\psi : \mathbb{R} \rightarrow \mathbb{C}$$



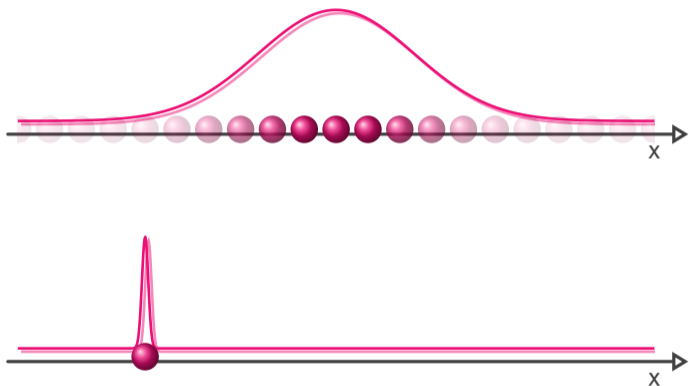
Wave-function collapse:

$$\Pr[\text{measured at } x] = |\psi(x)|^2$$



Probability distribution over possible outcomes

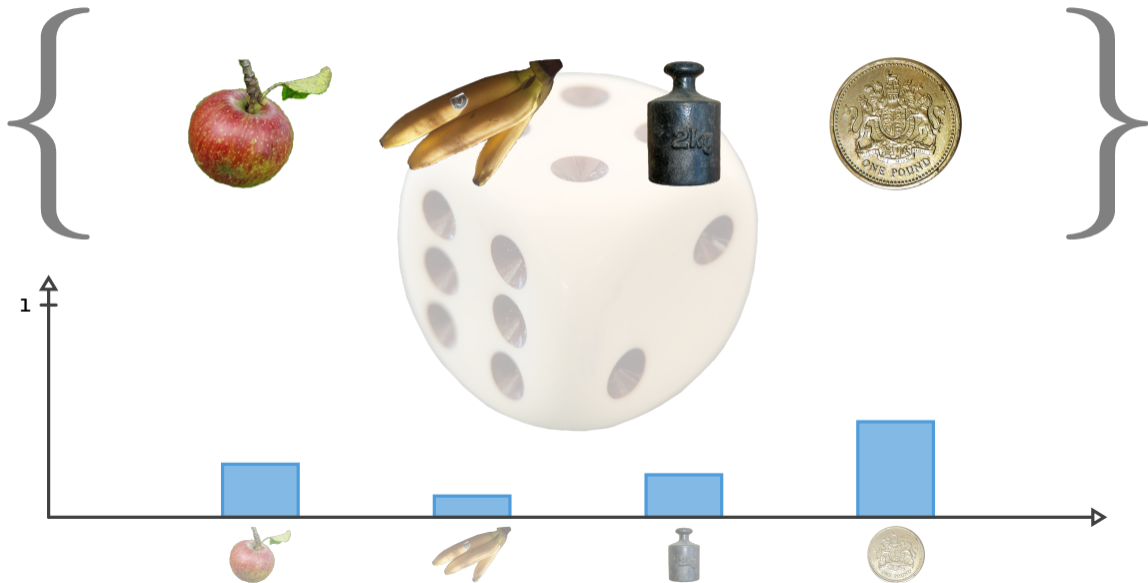
Sampling



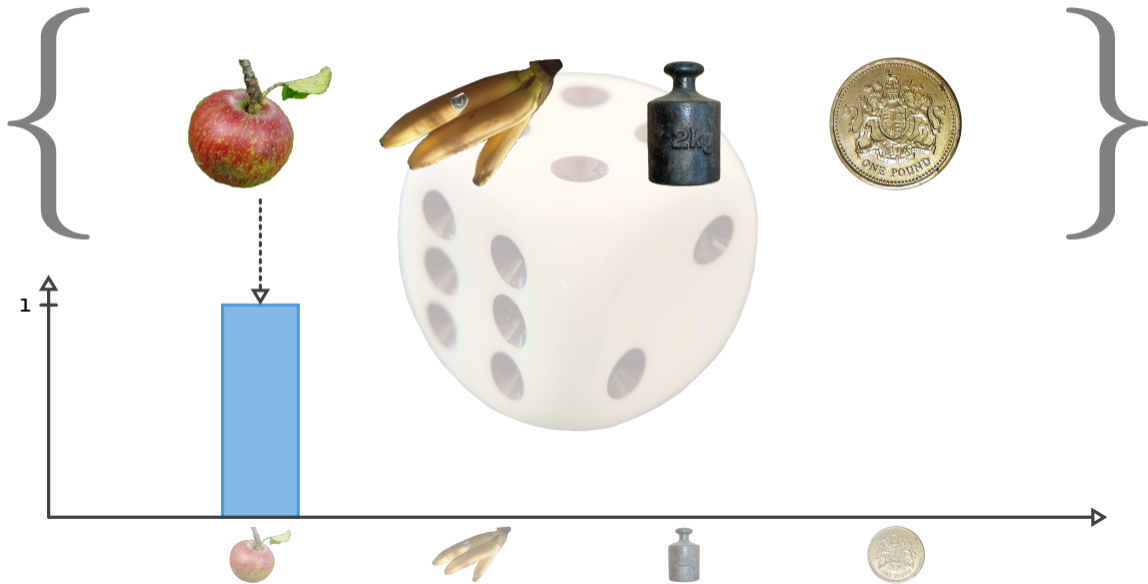
Sampling



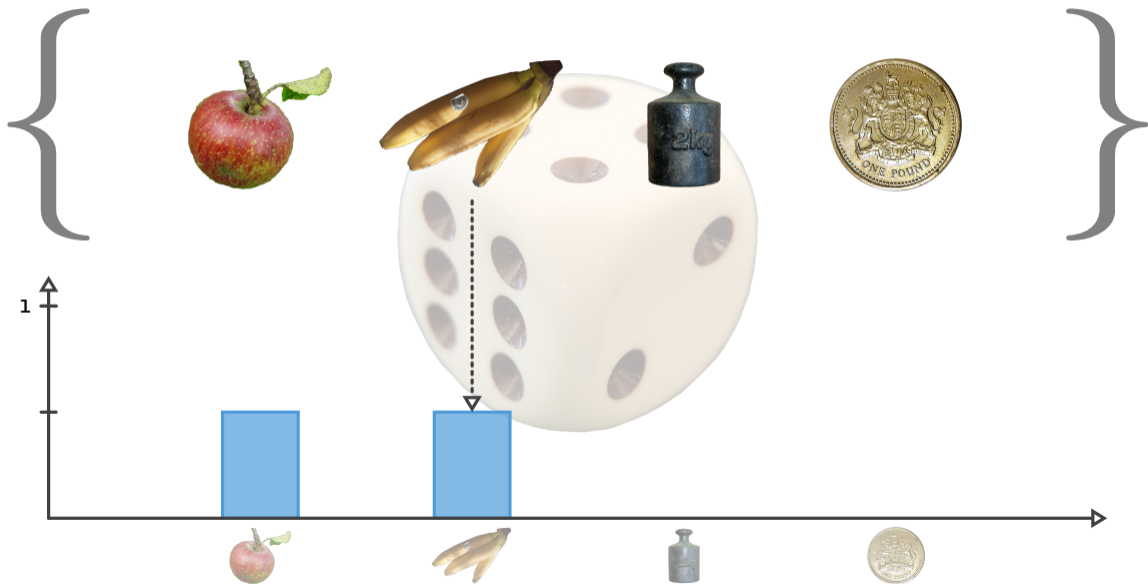
Sampling



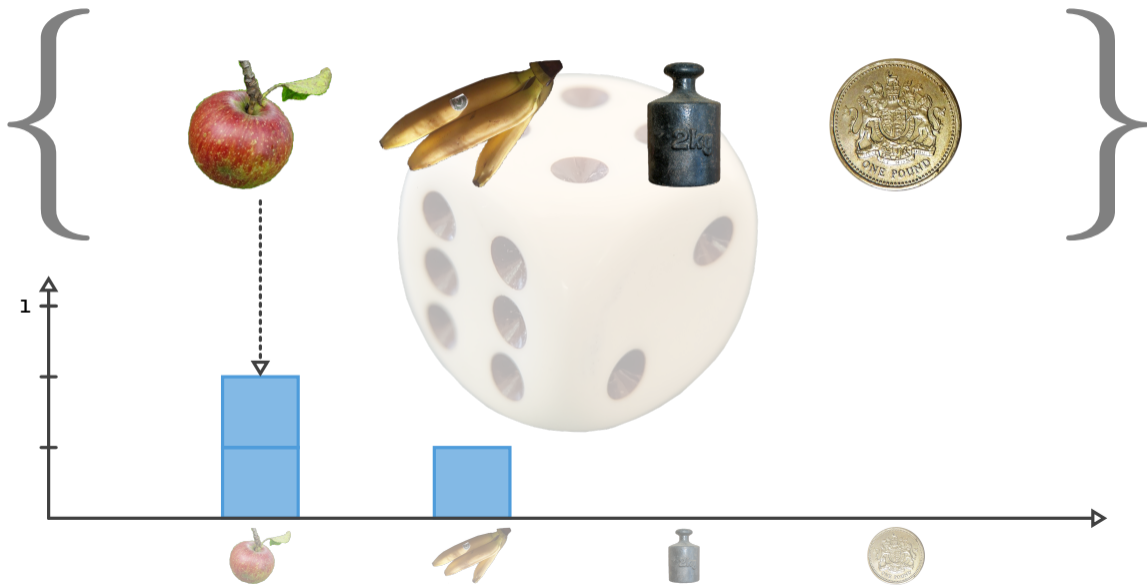
Sampling



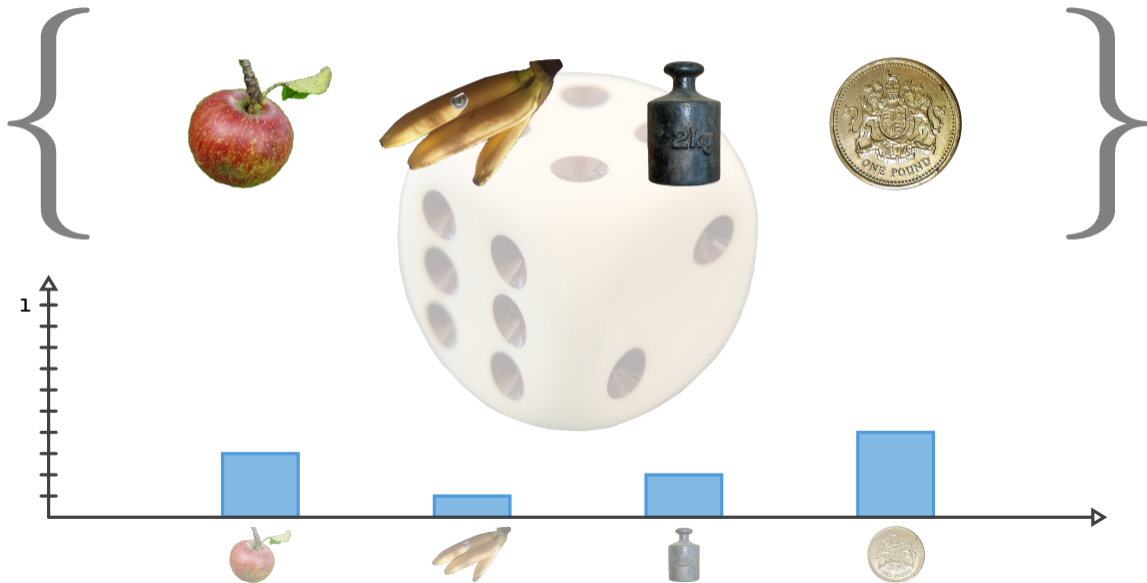
Sampling



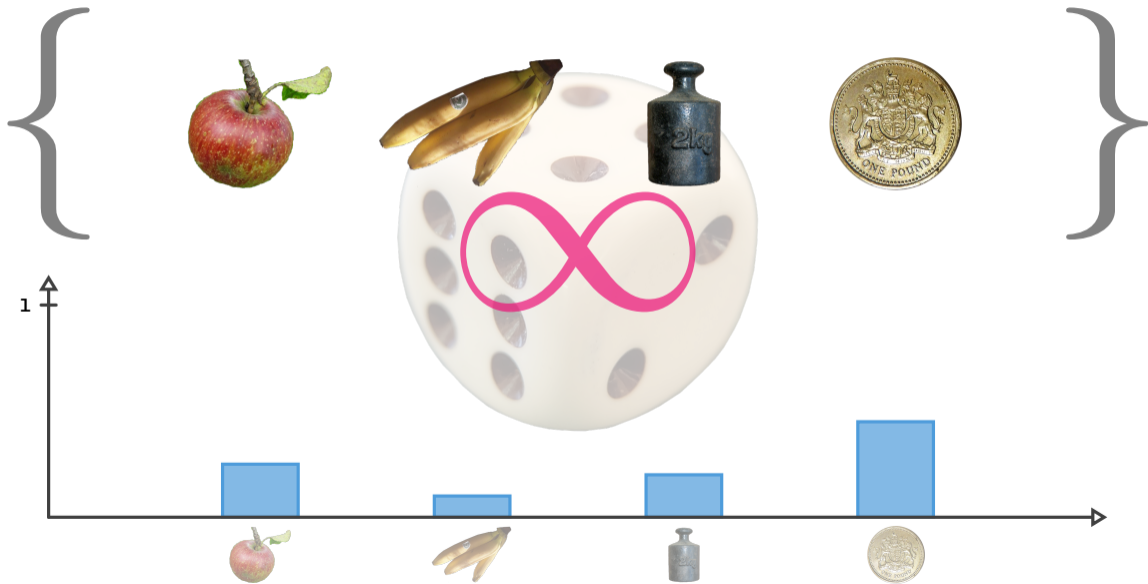
Sampling



Sampling



Sampling



Computational complexity: scaling of runtime



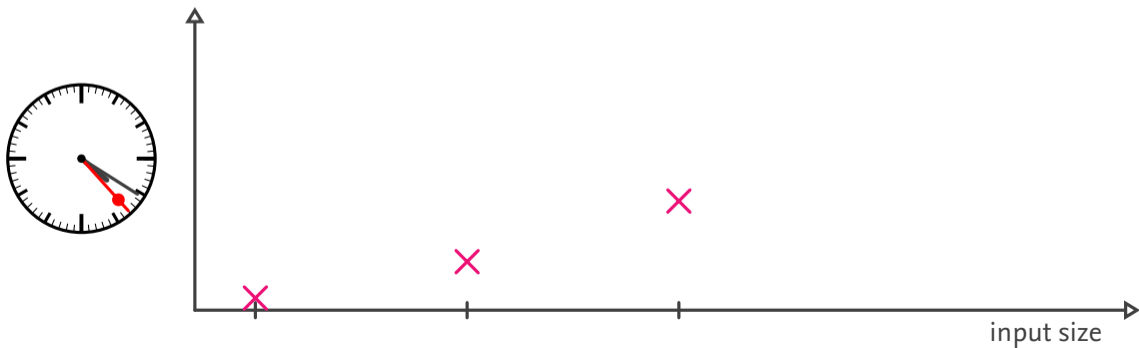
Computational complexity: scaling of runtime



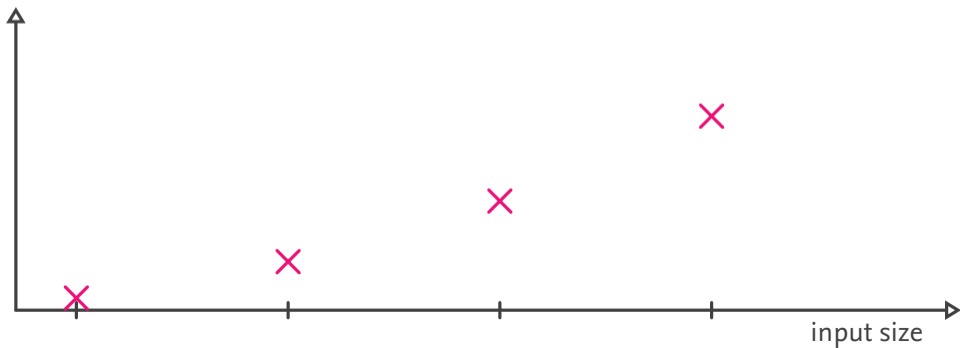
Computational complexity: scaling of runtime



Computational complexity: scaling of runtime



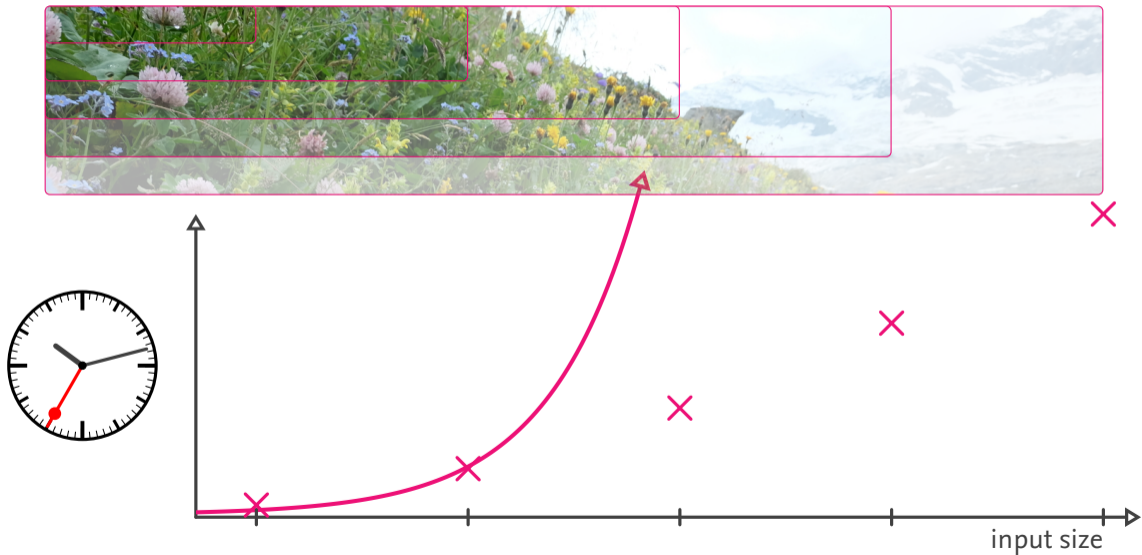
Computational complexity: scaling of runtime



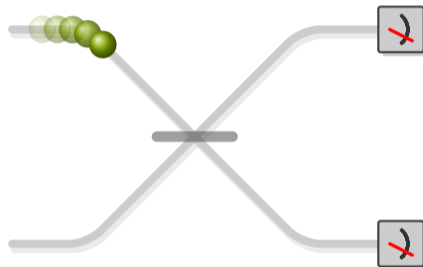
Computational complexity: scaling of runtime



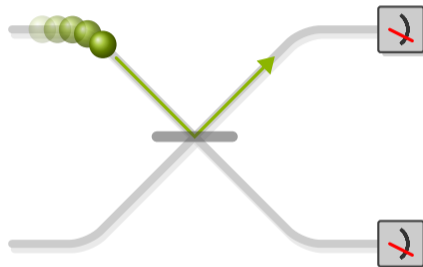
Computational complexity: scaling of runtime



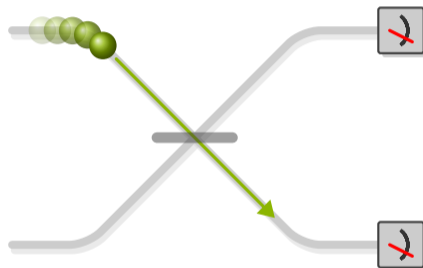
Quantum sampling: The Hong-Ou-Mandel interferometer



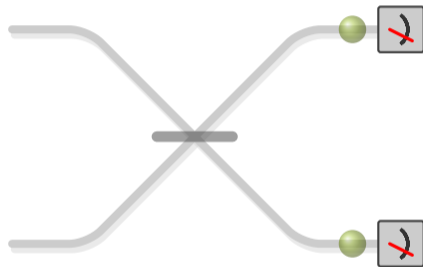
Quantum sampling: The Hong-Ou-Mandel interferometer



Quantum sampling: The Hong-Ou-Mandel interferometer

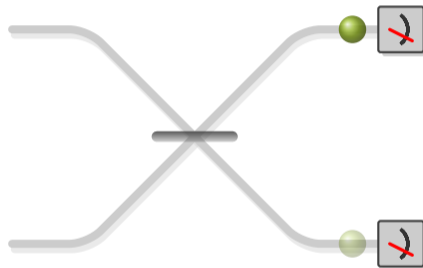


Quantum sampling: The Hong-Ou-Mandel interferometer



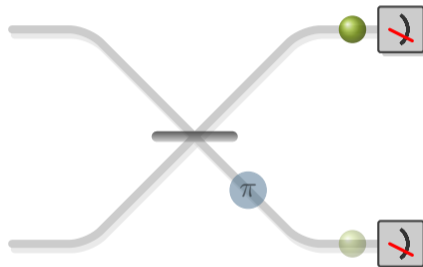
$$\frac{1}{\sqrt{2}} (|0, 1\rangle + |1, 0\rangle)$$

Quantum sampling: The Hong-Ou-Mandel interferometer



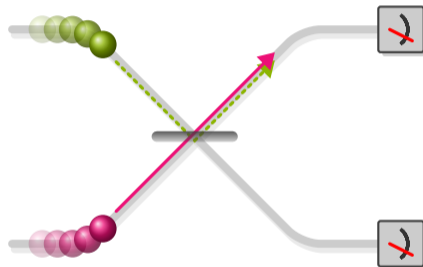
$$\sqrt{\frac{3}{5}}|0, 1\rangle + \sqrt{\frac{2}{5}}|1, 0\rangle$$

Quantum sampling: The Hong-Ou-Mandel interferometer

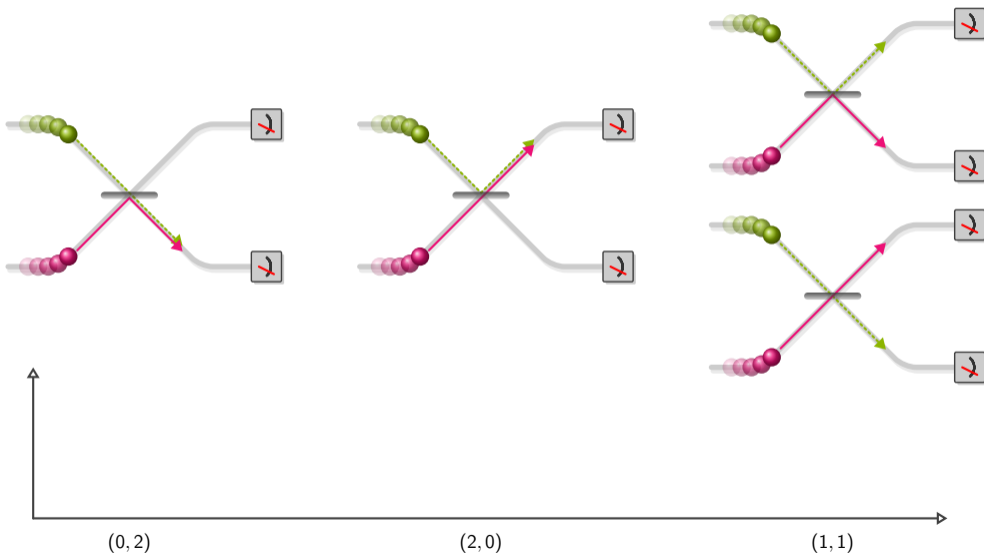


$$\sqrt{\frac{3}{5}}|0, 1\rangle - \sqrt{\frac{2}{5}}|1, 0\rangle$$

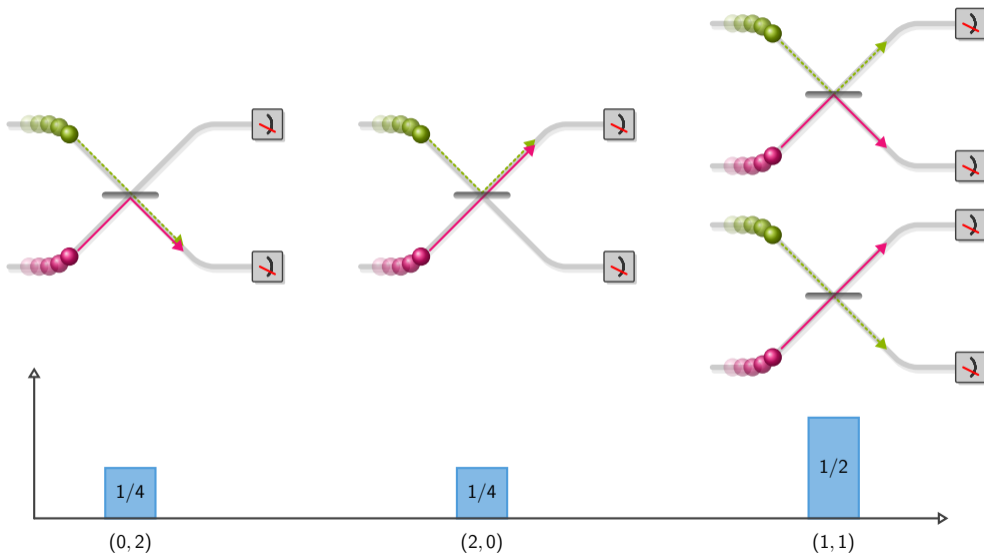
Quantum sampling: The Hong-Ou-Mandel interferometer



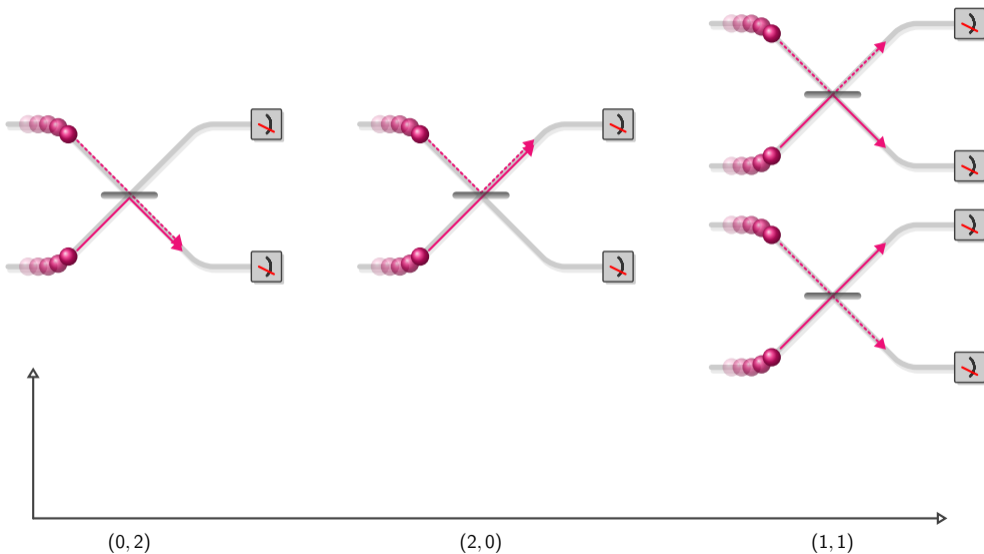
Quantum sampling: The Hong-Ou-Mandel interferometer – two photons



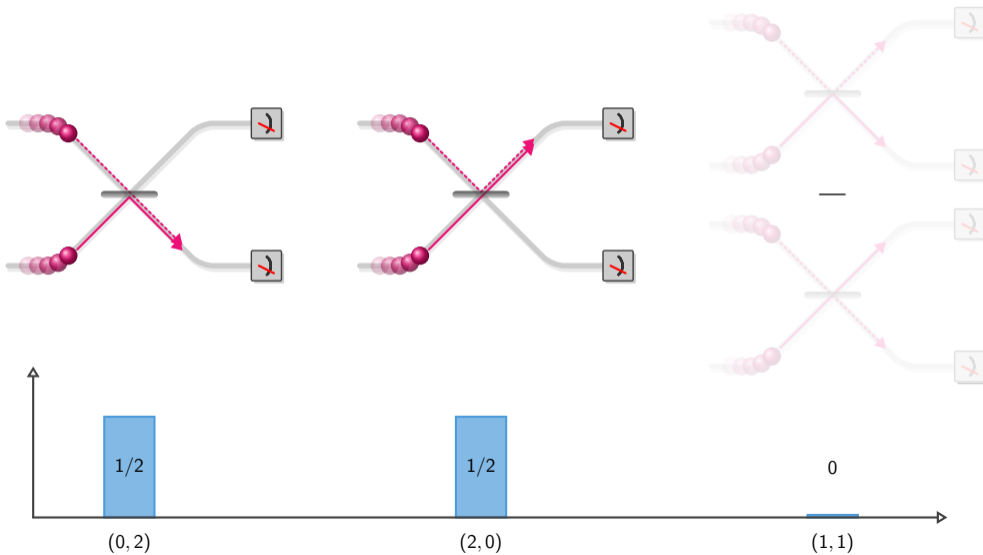
Quantum sampling: The Hong-Ou-Mandel interferometer – two photons



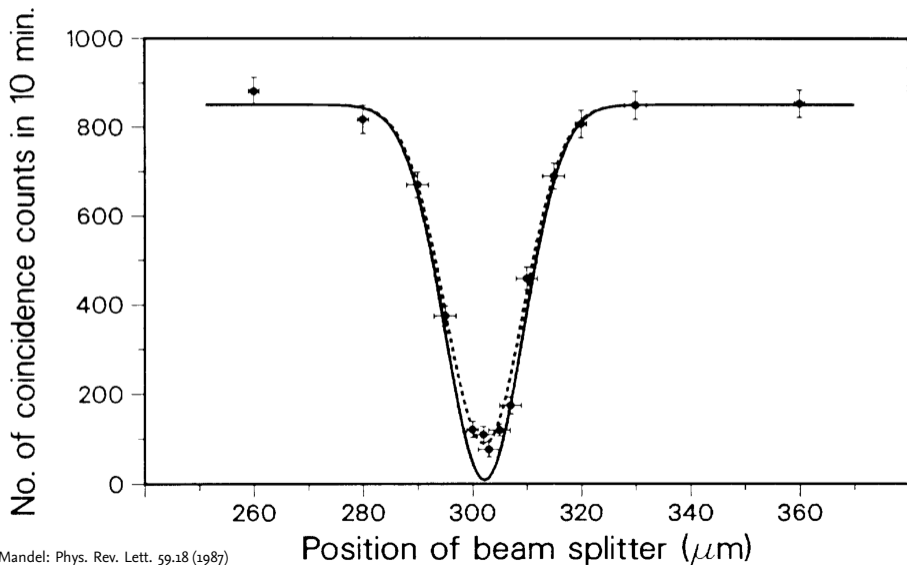
Quantum sampling: The Hong-Ou-Mandel interferometer – two photons



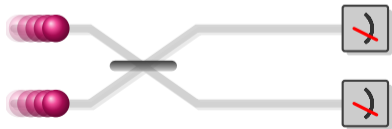
Quantum sampling: The Hong-Ou-Mandel interferometer – two photons



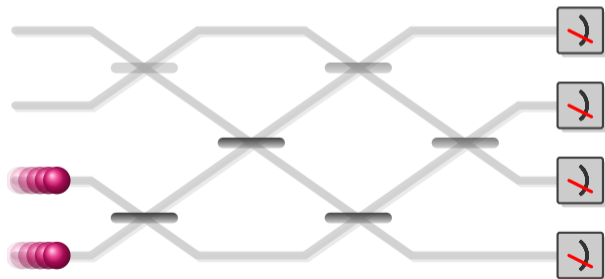
Quantum sampling: The Hong-Ou-Mandel interferometer – two photons



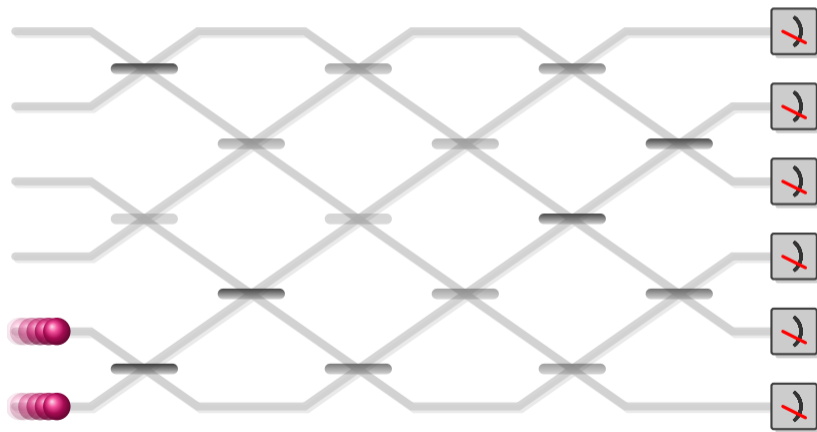
Complex quantum sampling: A large network of HOM interferometers



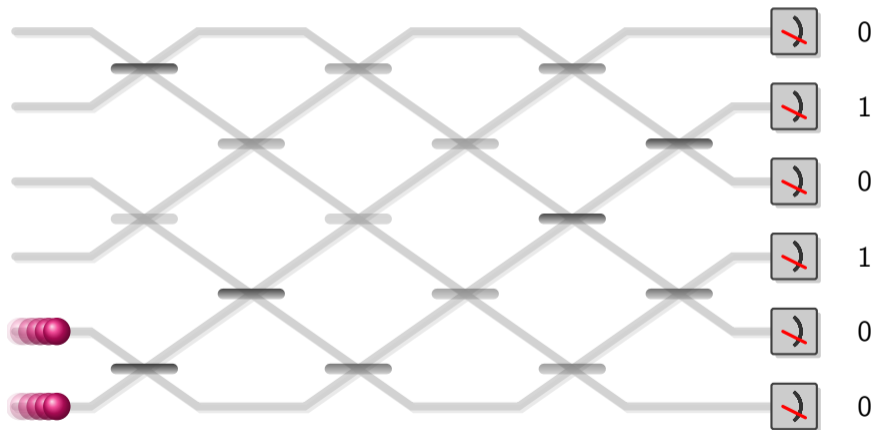
Complex quantum sampling: A large network of HOM interferometers



Complex quantum sampling: A large network of HOM interferometers

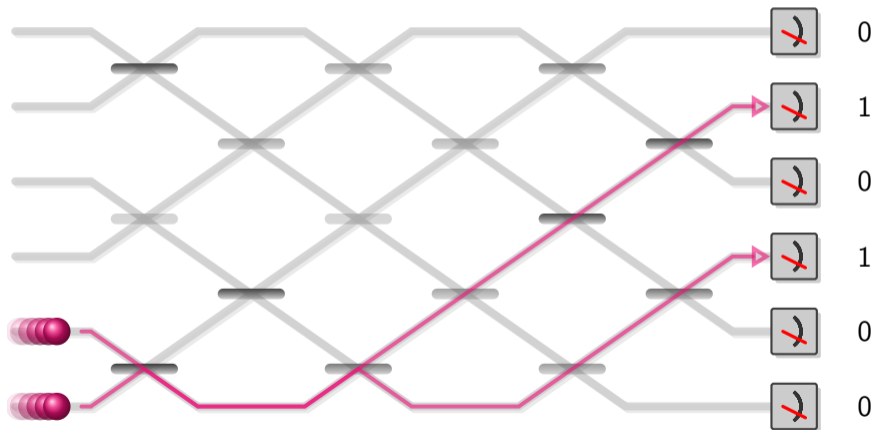


Complex quantum sampling: A large network of HOM interferometers

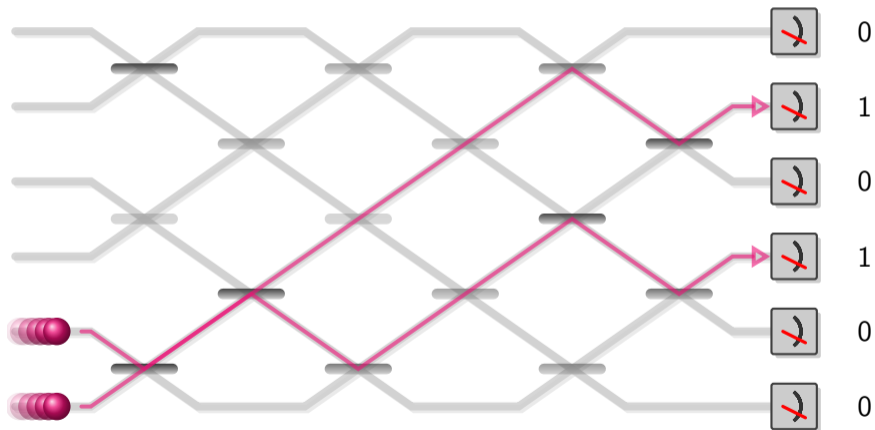


$$|\Phi_{m,n}| \sim m^n$$

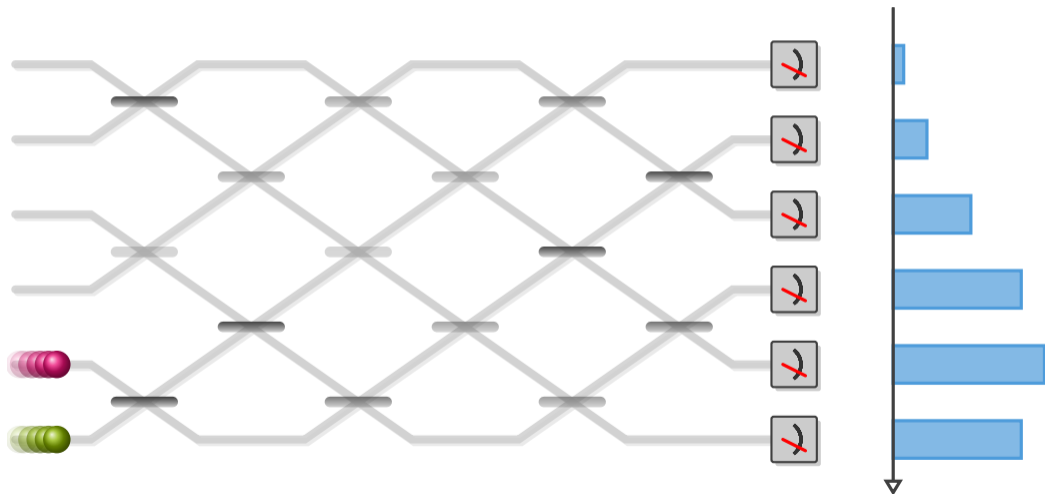
Complex quantum sampling: A large network of HOM interferometers



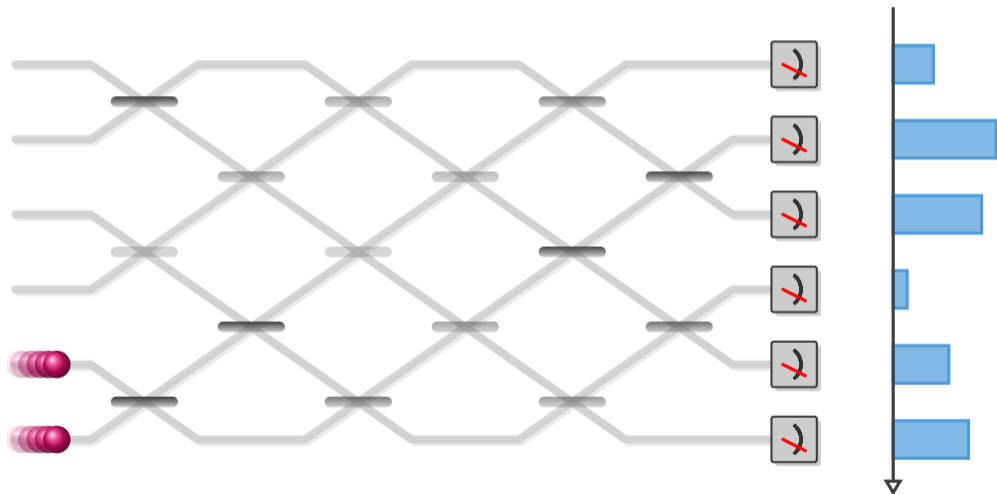
Complex quantum sampling: A large network of HOM interferometers



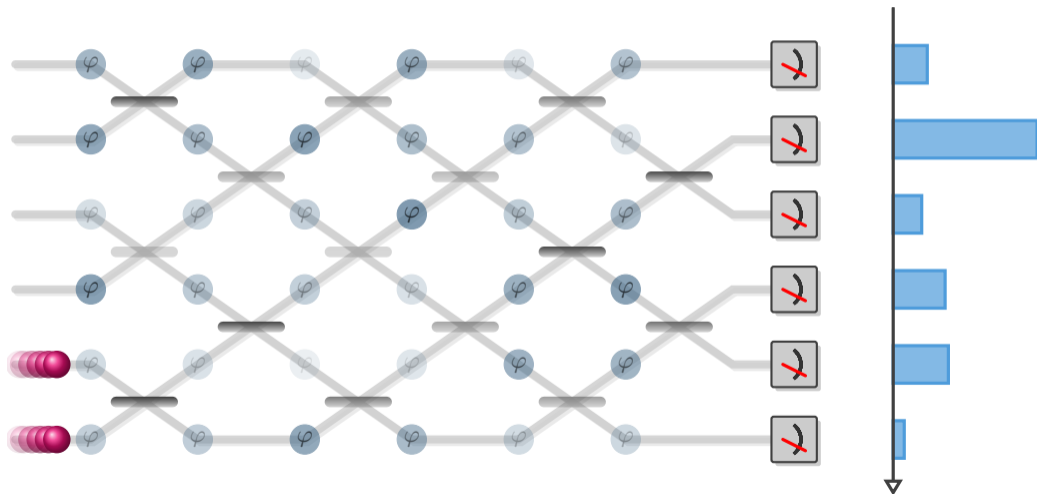
Complex quantum sampling: A large network of HOM interferometers



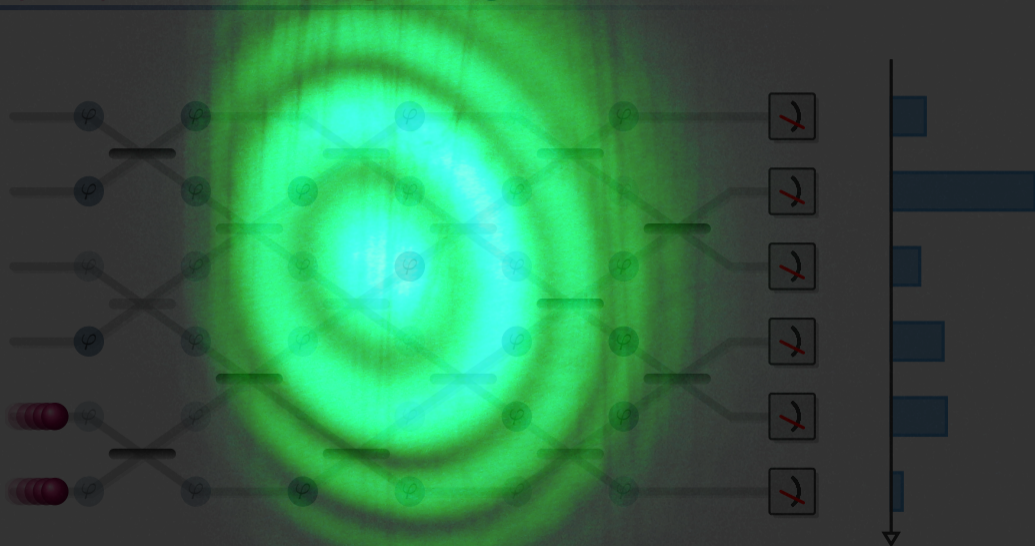
Complex quantum sampling: A large network of HOM interferometers



Complex quantum sampling: A large network of HOM interferometers



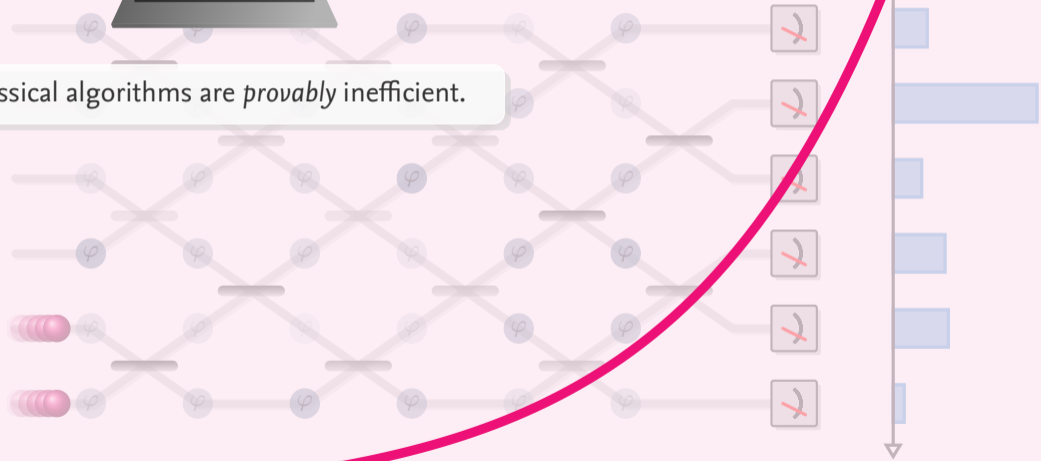
Complex quantum sampling: A large network of HOM interferometers



Complex quantum sampling: A large network of HOM interferometers



Classical algorithms are *provably* inefficient.

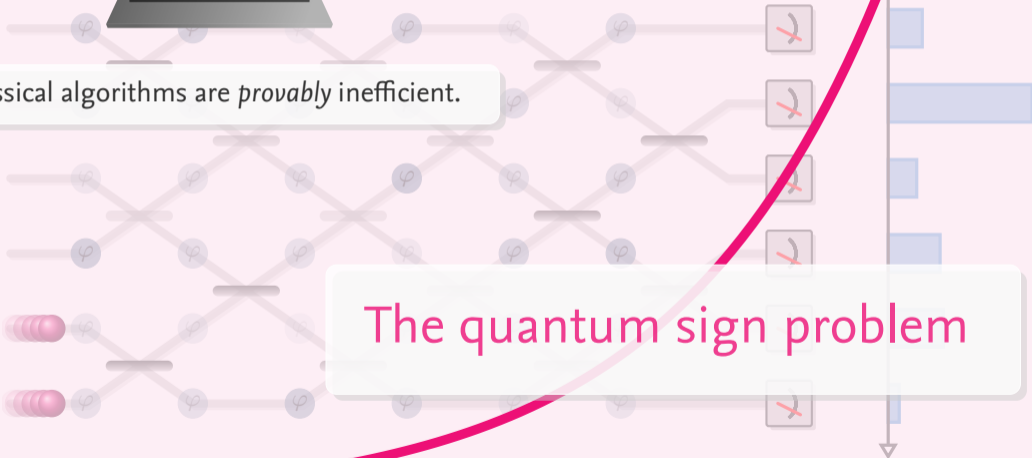


Complex quantum sampling: A large network of HOM interferometers

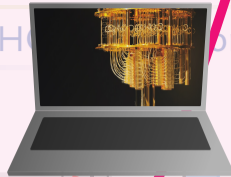


Classical algorithms are *provably* inefficient.

The quantum sign problem

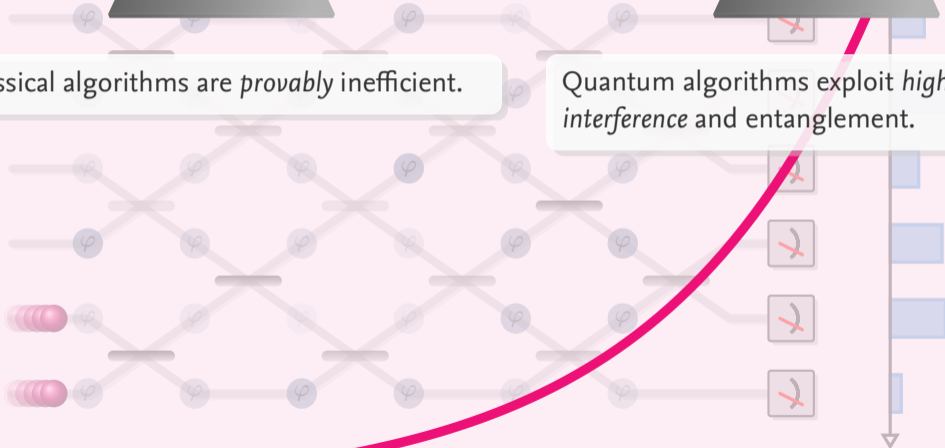


Complex quantum simulation: A large network of Hadamard gates and CNOTs meters

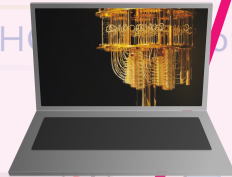


Classical algorithms are *provably* inefficient.

Quantum algorithms exploit *high-dimensional interference* and entanglement.



Complex quantum sampling: A large network of Hadamard gates



Classical algorithms are *provably* inefficient.

Quantum algorithms exploit *high-dimensional interference* and entanglement.



How is the **quantum sign problem** reflected in the computational complexity of different sampling-related tasks?

Delineating the quantum-classical boundary



Classical

Quantum

Delineating the quantum-classical boundary

Classical

Quantum

1. Sampling hardness in generic quantum computations

Delineating the quantum-classical boundary

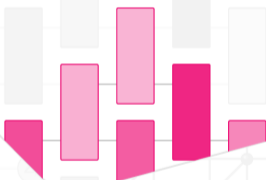
Quantum

1. Sampling hardness in generic quantum computations

Classical

2. Estimating outcome probabilities

Perspective 1
The sign problem in complexity theory



The sign problem in complexity theory: the basics

3-SAT formula

$$f(x) = (x_1 \vee \bar{x}_7 \vee x_{137}) \wedge (x_5 \vee x_{12} \vee \bar{x}_{17}) \wedge (\bar{x}_{32} \vee \bar{x}_7 \vee x_{17}) \wedge \cdots \wedge (\bar{x}_3 \vee \bar{x}_2 \vee \bar{x}_1)$$

NP: Decide whether $\exists x : f(x) = 1$.

#P: Compute $acc(f) := |\{x : f(x) = 1\}| \in \{0, \dots, 2^n\}$.

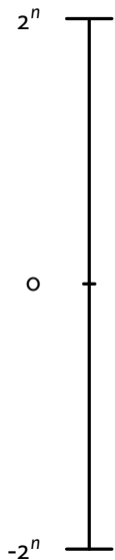
GapP: Compute $gap(f) := acc(f) - rej(f) \in \{-2^n, \dots, 2^n\}$.

PP: Decide whether $gap(f) > 0$ or ≤ 0 .



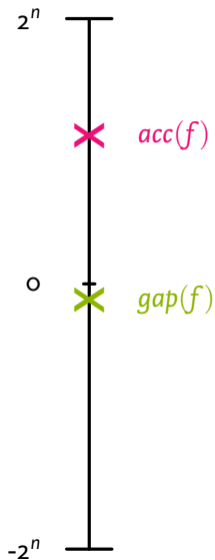
$$P^{\#P} = P^{\text{GapP}} = P^{\text{PP}}$$

The sign problem in complexity theory: approximations



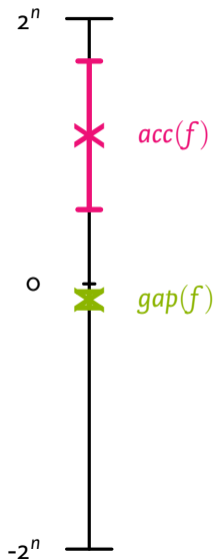
Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

The sign problem in complexity theory: approximations



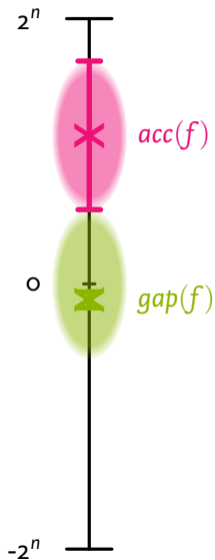
Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

The sign problem in complexity theory: approximations



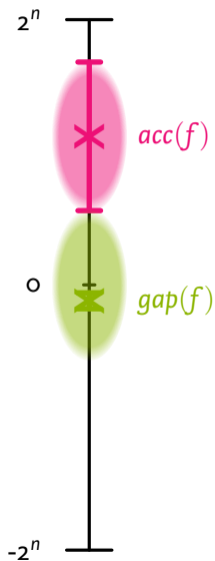
Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

The sign problem in complexity theory: approximations



Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

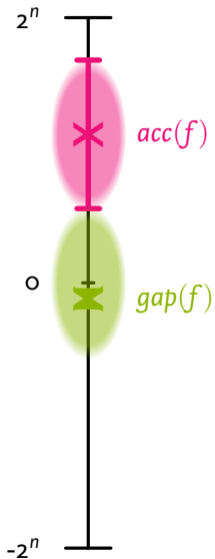
The sign problem in complexity theory: approximations



Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

$$P^{Apx_r \text{GapP}} = P^{\text{GapP}} = P^{\text{PP}}$$

The sign problem in complexity theory: approximations

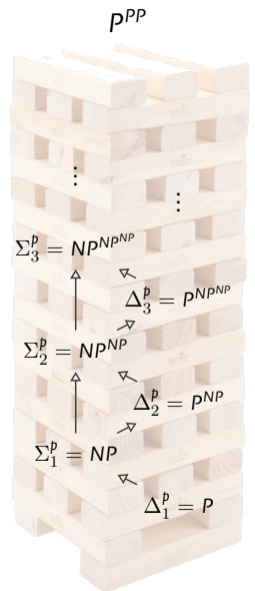


Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

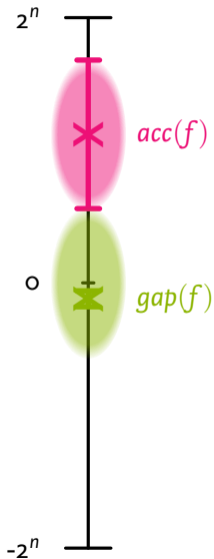
$$P^{Apx_r GapP} = P^{GapP} = P^{PP}$$

Stockmeyer '83: For any $r \in 1/\text{poly}(n)$:

$$Apx_r \#P \subset FBPP^{NP}$$



The sign problem in complexity theory: approximations

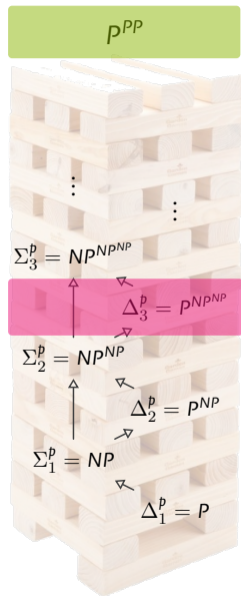


Relative error r : $(1 - r)X \leq \tilde{X} \leq (1 + r)X$

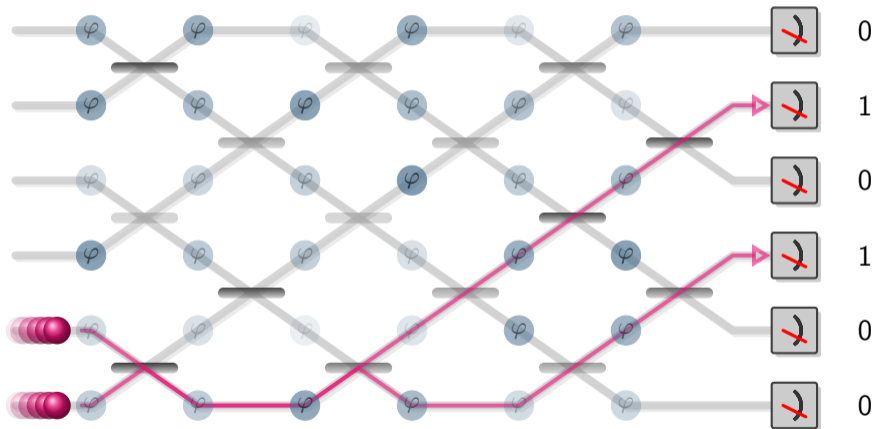
$$P^{Apx_r \text{GapP}} = P^{\text{GapP}} = P^{\text{PP}}$$

Stockmeyer '83: For any $r \in 1/\text{poly}(n)$:

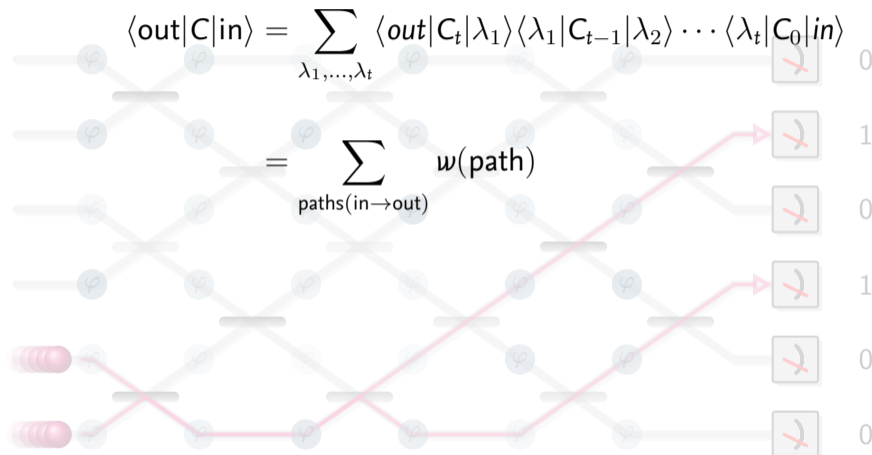
$$Apx_r \#P \subset \text{FBPP}^{\text{NP}}$$



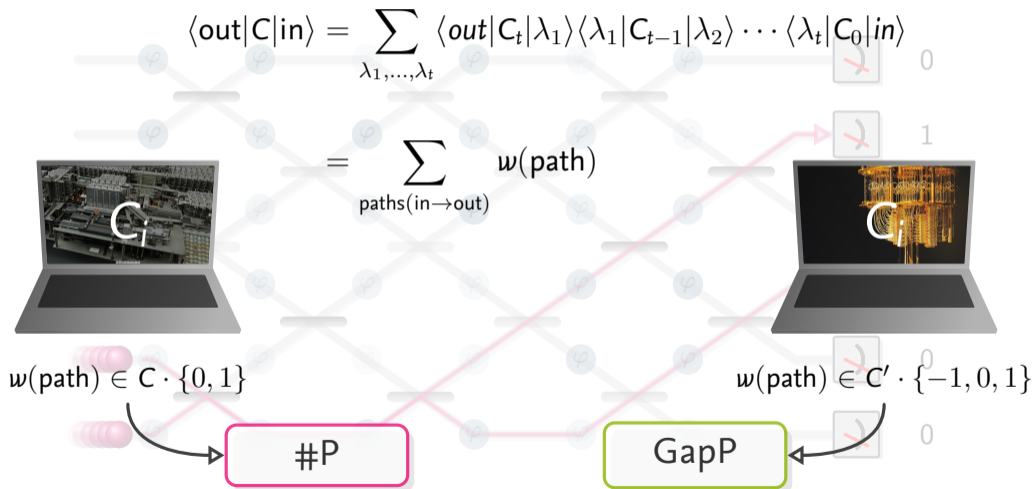
Success probabilities of quantum and classical circuits



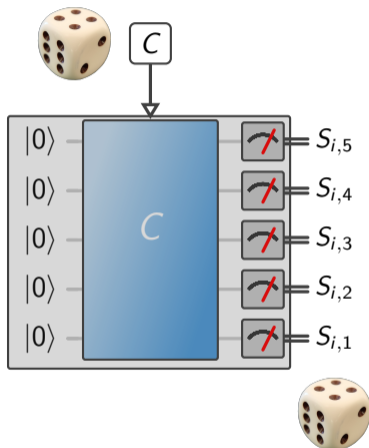
Success probabilities of quantum and classical circuits



Success probabilities of quantum and classical circuits



Leveraging hardness of estimation to hardness of sampling

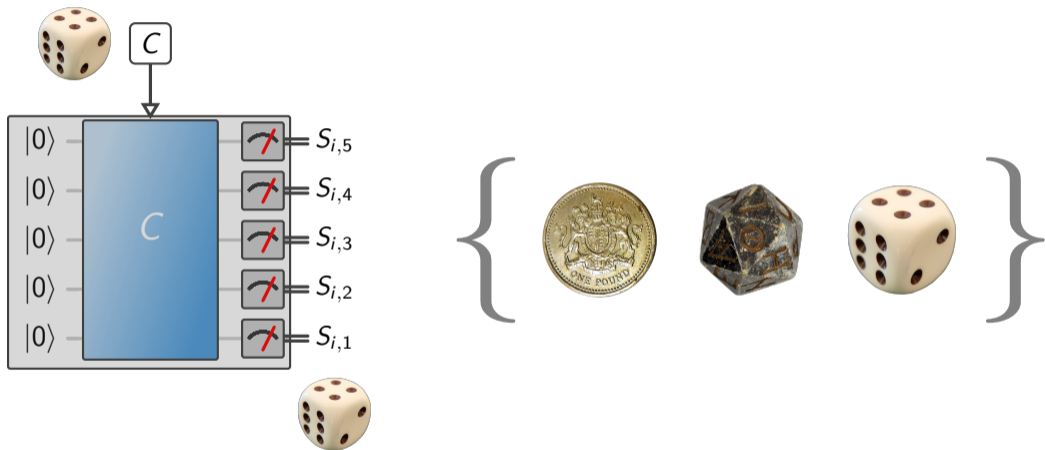


Consider

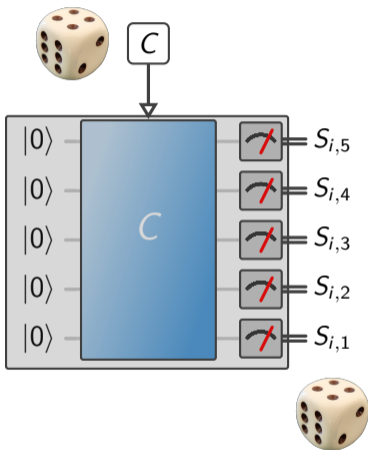
- n qubits
- \mathcal{U} family of quantum circuits, e.g.
 - \mathcal{U} = Universal circuits (e.g. using Clifford + T -gates) [Boi18]
 - \mathcal{U} = Diagonal (IQP) circuits [BMS16]
- $p_U(x) := |\langle x|U|0\rangle|^{\otimes n}|^2$

TASK: Given $U \in_R \mathcal{U}$, sample from p_U .

Leveraging hardness of estimation to hardness of sampling



Leveraging hardness of estimation to hardness of sampling



CLASSICAL DERANDOMIZABLE SAMPLING

Given $U \in \mathcal{U}$, uniformly random r , output y
s.t.

$$\Pr_x[y] \propto \sum_r f_U(r) = p_U(0^n),$$

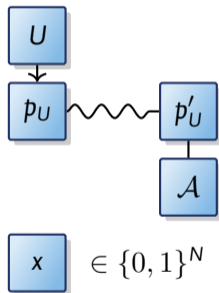
for $f_U : \{0, 1\}^m \rightarrow \{0, 1\}$ a #P function.

Approximate quantum sampling is classically hard

Assume there exists a *classical polynomial-time, derandomizable algorithm* \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.

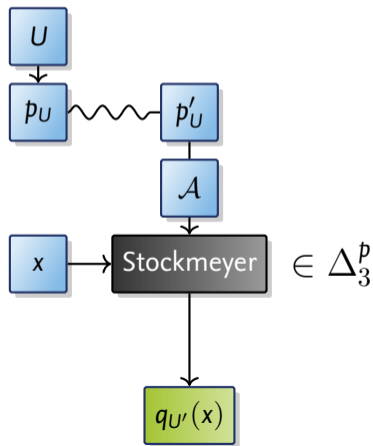
Approximate quantum sampling is classically hard

Assume there exists a *classical polynomial-time, derandomizable algorithm* \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



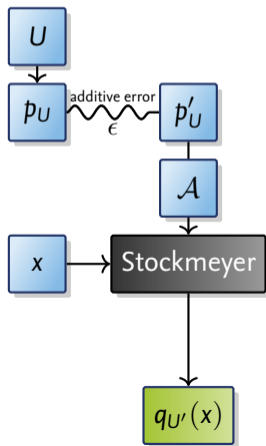
Approximate quantum sampling is classically hard

Assume there exists a *classical polynomial-time, derandomizable algorithm* \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



Approximate quantum sampling is classically hard

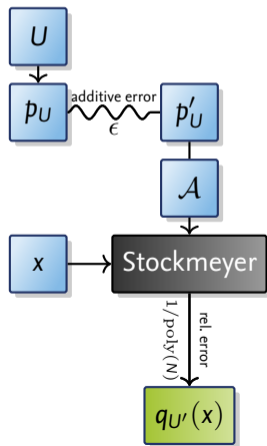
Assume there exists a *classical polynomial-time, derandomizable algorithm* \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



Approximate sampling: $\|p_U - p'_U\|_{\ell_1} \leq \epsilon$

Approximate quantum sampling is classically hard

Assume there exists a *classical polynomial-time, derandomizable algorithm* \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



Approximate sampling: $\|p_U - p'_U\|_{\ell_1} \leq \epsilon$

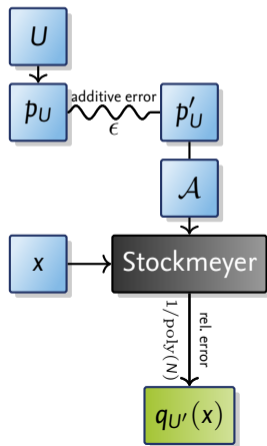
Stockmeyer error: $|q_U(x) - p'_U(x)| \leq \frac{p'_U(x)}{\text{poly}(n)}$

With probability $1 - \delta$ over $U \in_R \mathcal{U}$

$$|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)} + \frac{\epsilon}{2^n \delta} \left(1 + \frac{1}{\text{poly}(n)} \right)$$

Approximate quantum sampling is classically hard

Assume there exists a *classical polynomial-time, derandomizable algorithm* \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



Approximate sampling: $\|p_U - p'_U\|_{\ell_1} \leq \epsilon$

Stockmeyer error: $|q_U(x) - p'_U(x)| \leq \frac{p'_U(x)}{\text{poly}(n)}$

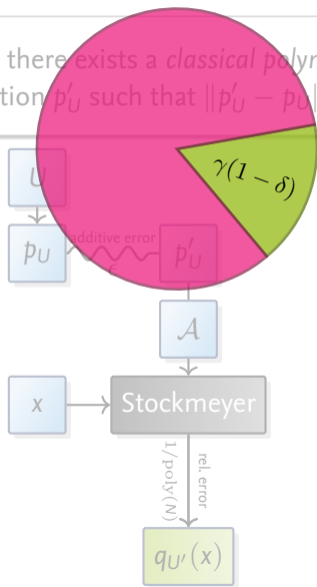
With probability $1 - \delta$ over $U \in_R \mathcal{U}$

$$|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)} + \frac{\epsilon}{2^n \delta} \left(1 + \frac{1}{\text{poly}(n)}\right)$$

$$\Pr_{U \in \mathcal{U}} [p_U(x) \geq \frac{\epsilon}{2^n \delta}] \geq \gamma$$

Approximate quantum sampling is classically hard

Assume there exists a classical polynomial-time, derandomizable algorithm \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



$$|q_U(x) - p_U(x)| \leq \text{const.} \cdot p_U(x)$$

Approximate sampling: $\|p_U - p'_U\|_{\ell_1} \leq \epsilon$

Stockmeyer error: $|q_U(x) - p'_U(x)| \leq \frac{p'_U(x)}{\text{poly}(n)}$

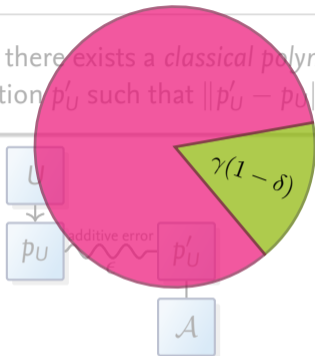
With probability $1 - \delta$ over $U \in_R \mathcal{U}$

$$|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)} + \frac{\epsilon}{2^n \delta} \left(1 + \frac{1}{\text{poly}(n)} \right)$$

$$\Pr_{U \in \mathcal{U}} [p_U(x) \geq \frac{\epsilon}{2^n \delta}] \geq \gamma$$

Approximate quantum sampling is classically hard

Assume there exists a classical polynomial-time, derandomizable algorithm \mathcal{A} that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



$$|q_U(x) - p_U(x)| \leq \text{const.} \cdot p_U(x)$$

Approximate sampling: $\|p_U - p'_U\|_{\ell_1} \leq \epsilon$

Stockmeyer error: $|q_U(x) - p'_U(x)| \leq \frac{p'_U(x)}{\text{poly}(n)}$

If $p_U(x)$ is **GapP-hard to estimate** for any $\gamma(1 - \delta)$ fraction of the instances, then ...

With probability $1 - \delta$ over $U \in_R \mathcal{U}$

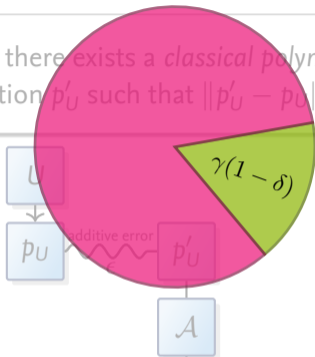
$$|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)} + \frac{\epsilon}{2^n \delta} \left(1 + \frac{1}{\text{poly}(n)}\right)$$

$$\Pr_{U \in \mathcal{U}} [p_U(x) \geq \frac{\epsilon}{2^n \delta}] \geq \gamma$$

$q_U(x)$

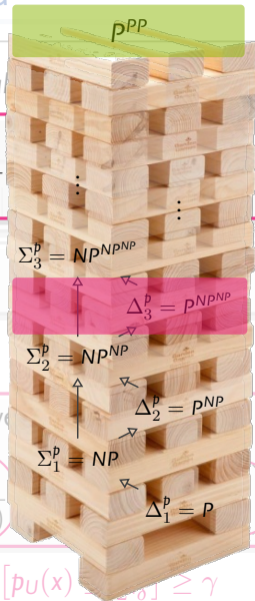
Approximate quantum sampling is classically hard

Assume there exists a classical polynomial-time, derandomizable algorithm A that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.



Approximate sampling: $|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)}$

Stockmeyer error: $|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)}$



es from a

If $p_U(x)$ is **GapP-hard to estimate** for any $\gamma(1 - \delta)$ fraction of the instances, then ...

With probability $1 - \delta$ over U , $|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)}$

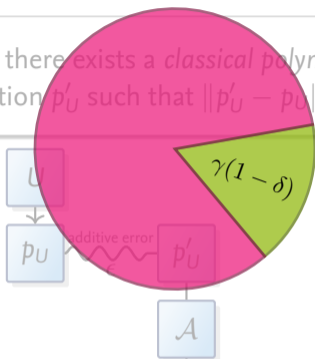
$q_{U'}(x)$

$$|q_U(x) - p_U(x)| \leq \frac{p_U(x)}{\text{poly}(n)}$$

$$\Pr_{U \in U} [p_U(x) \geq \gamma] \geq \gamma$$

Approximate quantum sampling is classically hard

Assume there exists a classical polynomial-time, derandomizable algorithm that samples from a distribution p'_U such that $\|p'_U - p_U\|_{\ell_1} \leq \epsilon$.

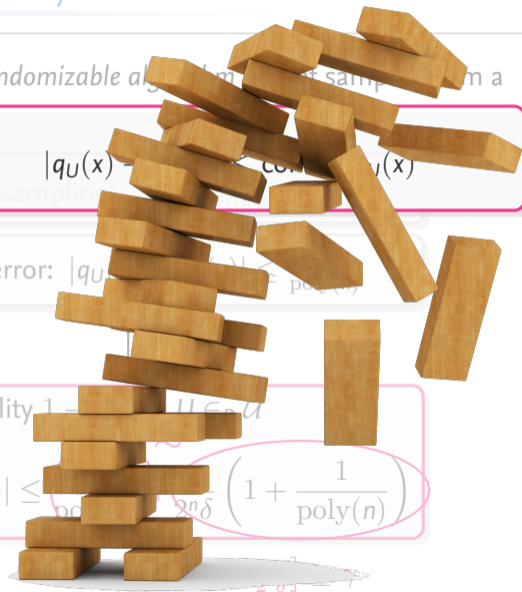


Approximate sampling

Stockmeyer error: $|q_U(x) - p_U(x)| \leq \text{poly}(n)$

If $p_U(x)$ is **GapP-hard to estimate** for any $\gamma(1 - \delta)$ fraction of the instances, then ...

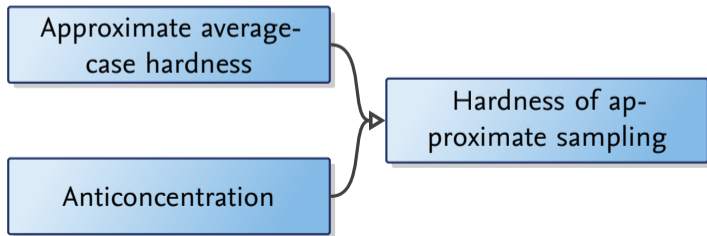
$q_U(x)$



With probability $1 - \epsilon$, $\|q_U - p_U\|_{\ell_1} \leq \epsilon$

$$|q_U(x) - p_U(x)| \leq \text{poly}(n) \left(1 + \frac{1}{\text{poly}(n)}\right)$$

Closing loopholes: the complexity-theoretic argument



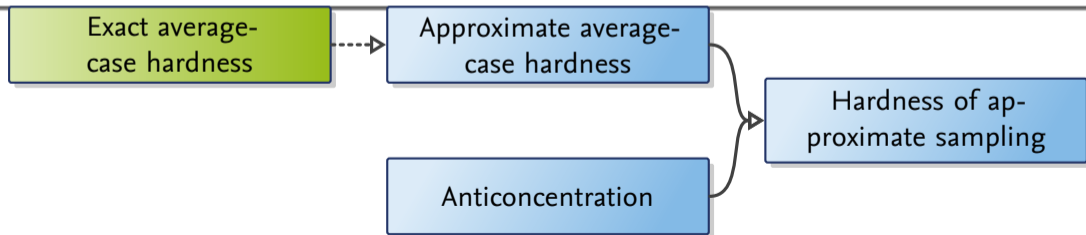
Closing loopholes: the complexity-theoretic argument

Theorem (Approximate worst-case hardness)

It is $\#P$ -hard to *approximate* the output probabilities of the RQIS scheme to within *relative error* $1/4$.

Theorem (Exact average-case hardness)

It is $\#P$ -hard to *exactly compute* any $3/4 + 1/\text{poly}(n)$ fraction of the output probabilities of the RQIS scheme.



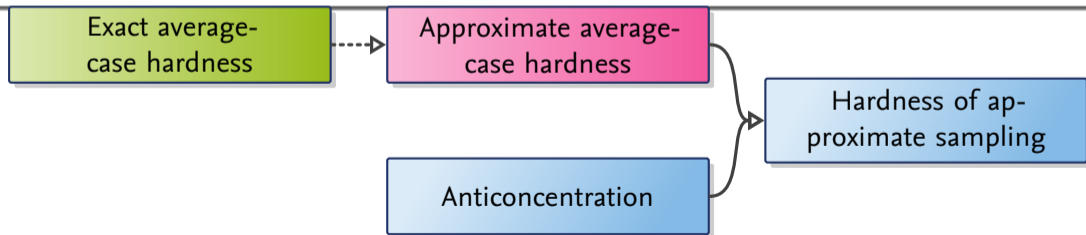
Closing loopholes: the complexity-theoretic argument

Theorem (Approximate worst-case hardness)

It is $\#P$ -hard to *approximate* the output probabilities of the RQIS scheme to within *relative error* $1/4$.

Theorem (Exact average-case hardness)

It is $\#P$ -hard to *exactly compute* any $3/4 + 1/\text{poly}(n)$ fraction of the output probabilities of the RQIS scheme.

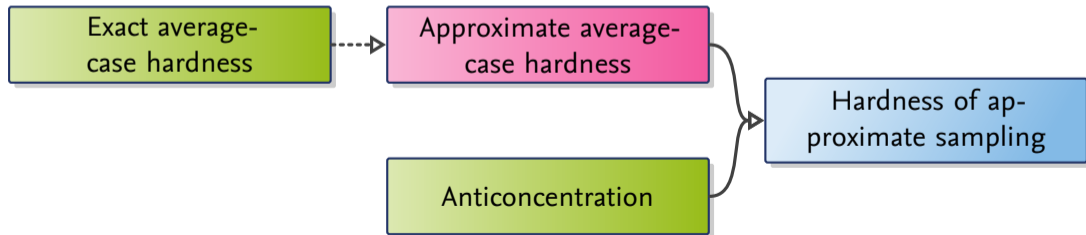


Closing loopholes: the complexity-theoretic argument

Theorem (Anticoncentration)

If a circuit family \mathcal{U} satisfies a second moment bound, then its output probabilities $|\langle x|U|0\rangle|^2$ anticoncentrate in the following sense: There exist constants $\alpha, \gamma(\alpha)$ such that

$$\Pr_{U \in_{\text{rand}} \mathcal{U}} \left[|\langle x|U|0\rangle|^2 > \frac{\alpha}{2^n} \right] \geq \gamma(\alpha)$$

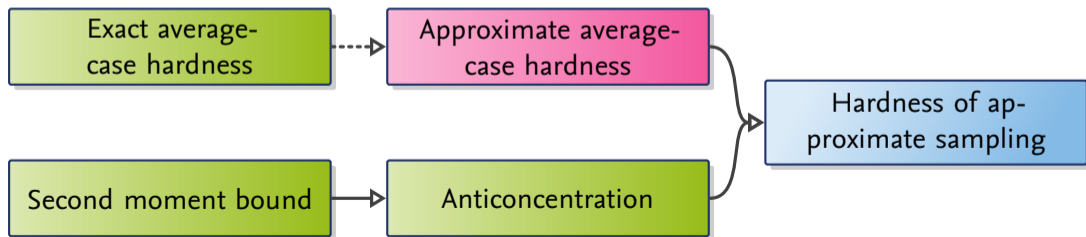


Closing loopholes: the complexity-theoretic argument

Theorem (Anticoncentration)

If a circuit family \mathcal{U} satisfies a second moment bound, then its output probabilities $|\langle x|U|0\rangle|^2$ anticoncentrate in the following sense: There exist constants $\alpha, \gamma(\alpha)$ such that

$$\Pr_{U \in_{\text{rand}} \mathcal{U}} \left[|\langle x|U|0\rangle|^2 > \frac{\alpha}{2^n} \right] \geq \gamma(\alpha) \iff \Pr[Z \geq \alpha \mathbb{E}[Z]] \geq (1 - \alpha)^2 \frac{\mathbb{E}[Z]^2}{\mathbb{E}[Z^2]}$$



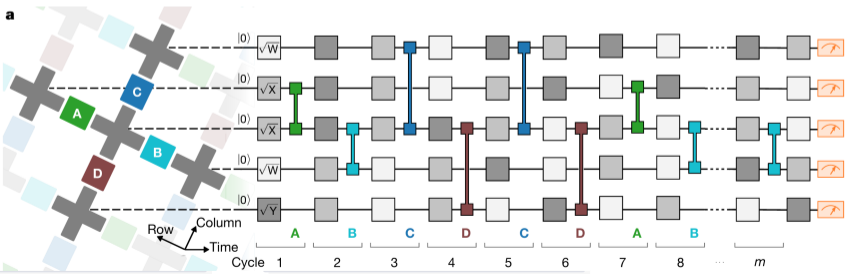
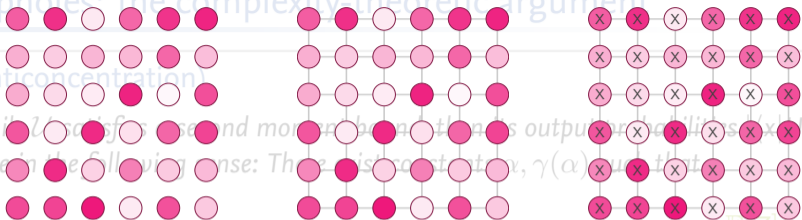
Closing loopholes: the complexity-theoretic argument

Theorem (Anticoncentration)

If a circuit family \mathcal{U} satisfies second moment bound then its output probabilities $\langle x | \mathcal{U} | 0 \rangle^2$ anticoncentrate in the following sense: There exist constant $\alpha, \gamma(\alpha)$ such that

$$\Pr_{U \in \text{rand } \mathcal{U}} \left[|\langle x | \mathcal{U} | 0 \rangle^2| > \frac{\alpha}{Z} \right] \geq \gamma(\alpha)$$

E1: Prepare $|\psi_\beta\rangle$ E2: Quench with H E3: Measure in X



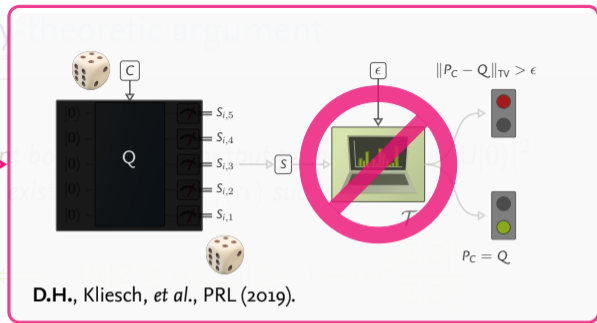
Sampling

Closing loopholes: the complexity-theoretic argument

Theorem (Anticoncentration)

If a circuit family \mathcal{U} satisfies a second moment bound, then the outputs of \mathcal{U} anticoncentrate in the following sense: There exist constants $\alpha, \gamma > 0$ such that

$$\Pr_{U \in \text{rand } \mathcal{U}} \left[|\langle x | U | 0 \rangle|^2 > \frac{\alpha}{2^n} \right] \geq \gamma(\alpha)$$



Exact average-case hardness

Approximate average-case hardness

Second moment bound

Anticoncentration

Hardness of approximate sampling

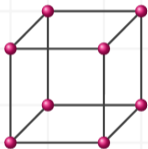
Delineating the quantum-classical boundary

Quantum

1. Sampling hardness in generic quantum computations

Classical

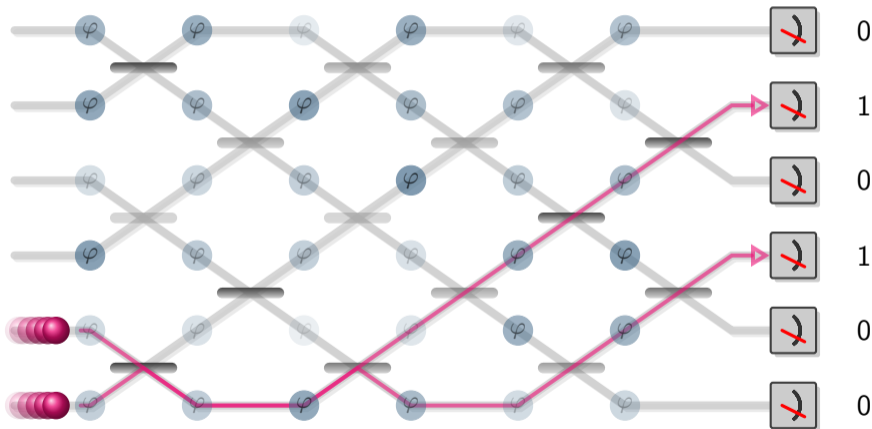
2. Estimating outcome probabilities



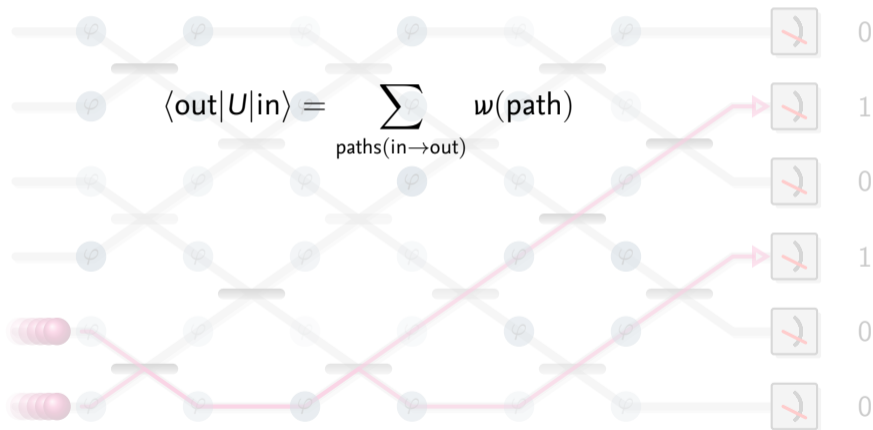
Perspective 2

Estimating quantum properties via classical sampling

The quantum sign problem

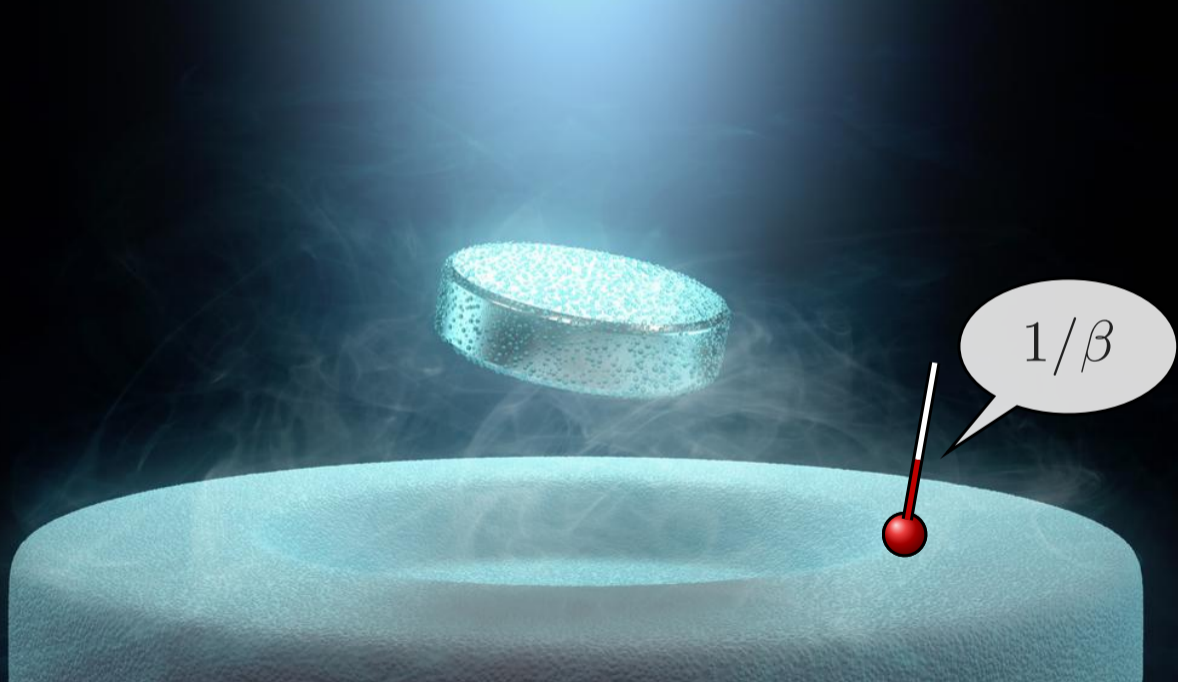


The quantum sign problem



Quantum Monte Carlo

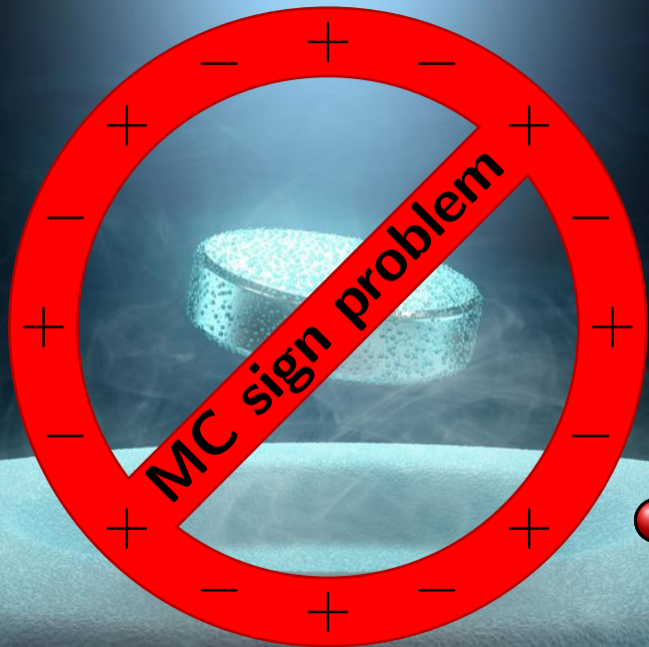




$$1/\beta$$



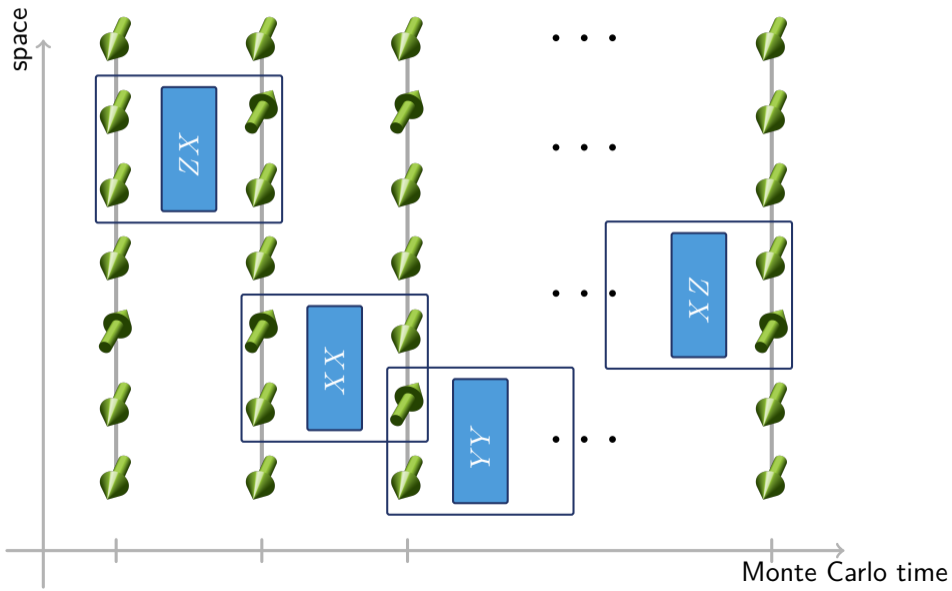
$1/\beta$



MC sign problem

$$1/\beta$$

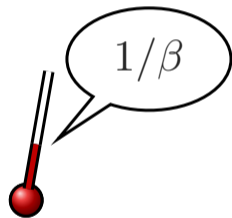
Quantum Monte Carlo



Quantum Monte Carlo

Calculating expectation values via expectation values

$$\langle O \rangle_{\beta, H} = \frac{1}{Z} \text{Tr}[e^{-\beta H} O] = \sum_{\lambda} q(\lambda) O(\lambda) = \langle O \rangle_q$$

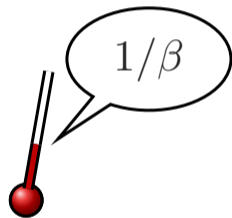


Quantum Monte Carlo

Calculating expectation values via expectation values

$$\langle O \rangle_{\beta, H} = \frac{1}{Z} \text{Tr}[e^{-\beta H} O] = \sum_{\lambda} q(\lambda) O(\lambda) = \langle O \rangle_q$$

$$e^{-\beta H} = \left(e^{-\beta H/m} \right)^m \approx \left(1 - \frac{\beta}{m} H \right)^m =: (T_m)^m$$



Quantum Monte Carlo

Calculating expectation values via expectation values

$$\langle O \rangle_{\beta, H} = \frac{1}{Z} \text{Tr}[e^{-\beta H} O] = \sum_{\lambda} q(\lambda) O(\lambda) = \langle O \rangle_q$$

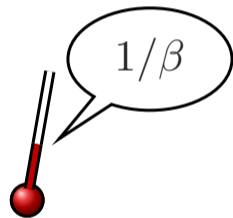
$$e^{-\beta H} = \left(e^{-\beta H/m} \right)^m \approx \left(1 - \frac{\beta}{m} H \right)^m =: (T_m)^m$$



$$\langle O \rangle_{\beta, H} = \frac{1}{Z} \text{Tr}[e^{-\beta H} O] \approx \frac{1}{Z} \text{Tr}[T_m^m O]$$

$$= \frac{1}{Z} \sum_{\lambda} T_m(\lambda_1 | \lambda_2) T_m(\lambda_2 | \lambda_3) \cdots T_m(\lambda_m | \lambda_{m+1}) O(\lambda_{m+1} | \lambda_1)$$

$$\equiv \frac{1}{Z} \sum_{\lambda} a(\lambda) O(\lambda)$$



Quantum Monte Carlo

Calculating expectation values via expectation values

$$\langle O \rangle_{\beta, H} = \frac{1}{Z} \text{Tr}[e^{-\beta H} O] = \sum_{\lambda} q(\lambda) O(\lambda) = \langle O \rangle_q$$

$$e^{-\beta H} = \left(e^{-\beta H/m} \right)^m \approx \left(1 - \frac{\beta}{m} H \right)^m =: (T_m)^m$$

$1/\beta$

Monte Carlo sampling

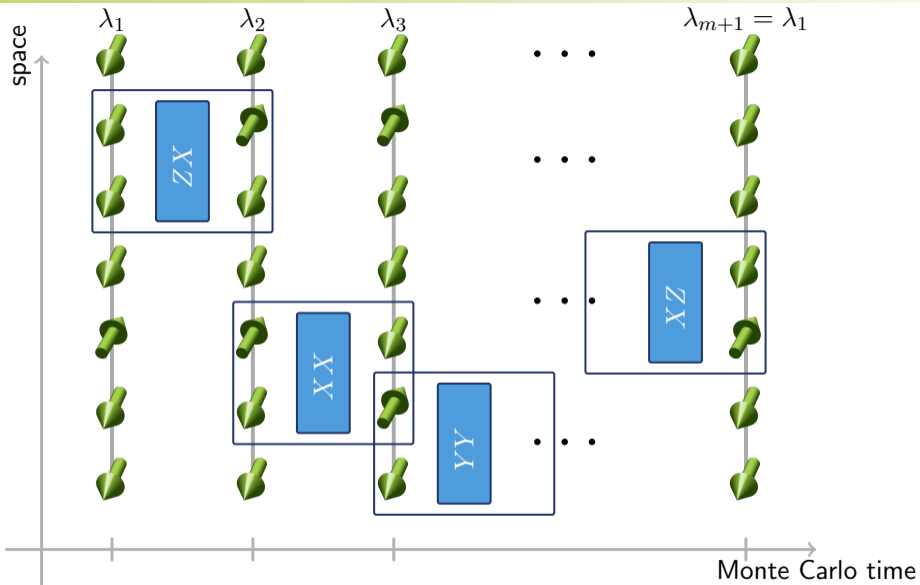
$\langle O \rangle_{\beta}$



Draw λ with probability $q(\lambda) = \frac{a(\lambda)}{\sum_{\lambda} a(\lambda)}$.

- **Metropolis sampling** with transition rate $W_{\lambda \rightarrow \lambda'} = q(\lambda')/q(\lambda)$:
- Markov chain: $\lambda^{(1)} \rightarrow \lambda^{(2)} \rightarrow \dots \rightarrow \lambda^{(s)}$

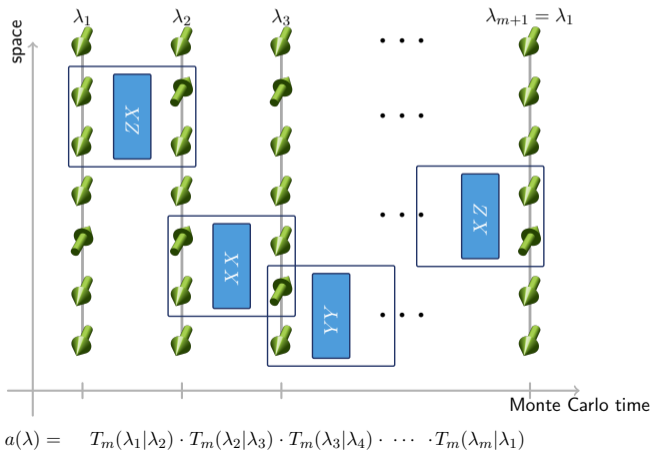
$$\equiv \frac{1}{Z} \sum_{\lambda} a(\lambda) O(\lambda)$$



$$a(\lambda) = T_m(\lambda_1|\lambda_2) \cdot T_m(\lambda_2|\lambda_3) \cdot T_m(\lambda_3|\lambda_4) \cdot \dots \cdot T_m(\lambda_m|\lambda_1)$$

The Monte Carlo sign problem

$$\langle O \rangle_{\beta, H} = \langle O \rangle_q$$



The Monte Carlo sign problem

$$\langle O \rangle_{\beta, H} = \langle O \rangle_q = \sum_{\lambda} p(\lambda) O'(\lambda)$$

The best Monte Carlo estimator: Absolute value

The variance-optimal estimator

$$q(\lambda) \rightarrow p(\lambda) = |q(\lambda)| / \|q(\lambda)\|_{\ell_1}$$

$$O(\lambda) \rightarrow O'(\lambda) = \text{sign}(q(\lambda)) \|q(\lambda)\|_{\ell_1} O(\lambda)$$

The Monte Carlo sign problem

$$\langle O \rangle_{\beta, H} = \langle O \rangle_q = \sum_{\lambda} p(\lambda) O'(\lambda) \approx \frac{1}{M} \frac{1}{\langle \text{sign} \rangle} \sum_{\lambda_1, \dots, \lambda_M} \text{sign}(q(\lambda_i))$$

The best Monte Carlo estimator: Absolute value

The variance-optimal estimator

$$q(\lambda) \rightarrow p(\lambda) = |q(\lambda)| / \|q(\lambda)\|_{\ell_1}$$

$$O(\lambda) \rightarrow O'(\lambda) = \text{sign}(q(\lambda)) \|q(\lambda)\|_{\ell_1} O(\lambda)$$



$\lambda_i \sim_{\text{rand}} p$

The Monte Carlo sign problem

$$\langle O \rangle_{\beta, H} = \langle O \rangle_q = \sum_{\lambda} p(\lambda) O'(\lambda) \approx \frac{1}{M} \frac{1}{\langle \text{sign} \rangle} \sum_{\lambda_1, \dots, \lambda_M} \text{sign}(q(\lambda_i)) \pm \frac{1}{\sqrt{M}} \frac{1}{\langle \text{sign} \rangle}$$

The best Monte Carlo estimator: Absolute value

The variance-optimal estimator

$$q(\lambda) \rightarrow p(\lambda) = |q(\lambda)| / \|q(\lambda)\|_{\ell_1}$$

$$O(\lambda) \rightarrow O'(\lambda) = \text{sign}(q(\lambda)) \|q(\lambda)\|_{\ell_1} O(\lambda)$$



$$\lambda_i \sim_{\text{rand}} p$$

has sample complexity

$$s \sim \|q\|_{\ell_1}^2 - 1 \equiv \frac{1}{\langle \text{sign} \rangle_q^2} - 1$$

$$\text{with } \langle \text{sign} \rangle_q = \frac{\sum_{\lambda} |q(\lambda)| \text{sign}(q(\lambda))}{\sum_{\lambda} |q(\lambda)|}.$$

The Monte Carlo sign problem

$$\langle O \rangle_{\beta, H} = \langle O \rangle_q = \sum_{\lambda} p(\lambda) O'(\lambda) \approx \frac{1}{M} \frac{1}{\langle \text{sign} \rangle} \sum_{\lambda_1, \dots, \lambda_M} \text{sign}(q(\lambda_i)) \pm \frac{1}{\sqrt{M}} \frac{1}{\langle \text{sign} \rangle}$$

The best Monte Carlo estimator: Absolute value

The variance-optimal estimator

$$q(\lambda) \rightarrow p(\lambda) = |q(\lambda)| / \|q(\lambda)\|_{\ell_1}$$

$$O(\lambda) \rightarrow O'(\lambda) = \text{sign}(q(\lambda)) \|q(\lambda)\|_{\ell_1} O(\lambda)$$



$\lambda_i \sim_{\text{rand}} p$

has sample complexity

$$s \sim \|q\|_{\ell_1}^2 - 1 \equiv \frac{1}{\langle \text{sign} \rangle_q^2} - 1 \in \exp(n) \quad \text{with} \quad \langle \text{sign} \rangle_q = \frac{\sum_{\lambda} |q(\lambda)| \text{sign}(q(\lambda))}{\sum_{\lambda} |q(\lambda)|}$$

The Monte Carlo sign problem

The Monte Carlo sign problem

The best Monte Carlo estimator: Absolute value

The variance-optimal estimator

$$q(\lambda) \rightarrow p(\lambda) = |q(\lambda)| / \|q(\lambda)\|_{\ell_1}$$

$$O(\lambda) \rightarrow O'(\lambda) = \text{sign}(q(\lambda)) \|q(\lambda)\|_{\ell_1} O(\lambda)$$



$$\lambda_i \sim_{\text{rand}} p$$

has sample complexity

$$s \sim \|q\|_{\ell_1}^2 - 1 \equiv \frac{1}{\langle \text{sign} \rangle_q^2} - 1 \in \exp(n) \quad \text{with} \quad \langle \text{sign} \rangle_q = \frac{\sum_{\lambda} |q(\lambda)| \text{sign}(q(\lambda))}{\sum_{\lambda} |q(\lambda)|}.$$

The Monte Carlo sign problem

The Monte Carlo sign problem

- $T_m = (1 - \beta H/m)$
- $a(\lambda) = T_m(\lambda_1|\lambda_2)T_m(\lambda_2|\lambda_3) \cdots T_m(\lambda_m|\lambda_1)$
 $\Rightarrow q(\lambda) = a(\lambda) / \sum_{\lambda} a(\lambda)$
- Sign problem: $q(\lambda) < 0$
 $\Rightarrow p(\lambda) = |q(\lambda)| / \sum_{\lambda} |q(\lambda)|$
- Exponential increase in sample complexity!

has sample complexity

$$s \sim \|q\|_{\ell_1}^2 - 1 \equiv \frac{1}{\langle \text{sign} \rangle_q^2} - 1 \in \exp(n) \quad \text{with} \quad \langle \text{sign} \rangle_q = \frac{\sum_{\lambda} |q(\lambda)| \text{sign}(q(\lambda))}{\sum_{\lambda} |q(\lambda)|}.$$

Easing the sign problem

Sign problem 2: Invariance under basis transformations

$$\text{Tr}[e^{-\beta H} \mathbf{O}] = \text{Tr}[\mathbf{Z}e^{-\beta H} \mathbf{O} \mathbf{Z}^{-1}] = \text{Tr}[(\mathbf{Z}T_m \mathbf{Z}^{-1})^m \mathbf{Z} \mathbf{O} \mathbf{Z}^{-1}].$$

→→ The sign problem is a **basis-dependent** property. ←←

Easing the sign problem

Sign problem 2: Invariance under basis transformations

$$\text{Tr}[e^{-\beta H} \mathbf{O}] = \text{Tr}[Z e^{-\beta H} \mathbf{O} Z^{-1}] = \text{Tr}[(Z T_m Z^{-1})^m Z \mathbf{O} Z^{-1}].$$

→→ The sign problem is a **basis-dependent** property. ←←



Bravyi, DiVincenzo, Oliveira, Terhal:
arXiv:quant-ph/0606140.
Bravyi, Terhal: arXiv:0806.1746.
Cubitt, Montanaro: arXiv:1311.3161.

Troyer and Wiese, PRL, 2008.
Hastings, J. Math. Phys (2015)

Marvian, Hen, Lidar, arXiv:1802.03408.
Klassen and Terhal, arXiv:1806.05405.
Klassen, Marvian *et al.*, arXiv:1906.08800.

Easing the sign problem

Sign problem 2: Invariance under basis transformations

$$\text{Tr}[e^{-\beta H} \mathbf{O}] = \text{Tr}[\mathbf{Z} e^{-\beta H} \mathbf{O} \mathbf{Z}^{-1}] = \text{Tr}[(\mathbf{Z} T_m \mathbf{Z}^{-1})^m \mathbf{Z} \mathbf{O} \mathbf{Z}^{-1}].$$

→→ The sign problem is a **basis-dependent** property. ←←

Easing the sign problem computationally

Given H , does there exist an **efficiently computable** U (a local circuit) such that $U H U^\dagger$ has a smaller sign problem than H ?

Bravyi, DiVincenzo, Oliveira, Terhal: arXiv:quant-ph/0606140.
 Bravyi, Terhal: arXiv:0806.1746.
 Cubitt, Montanaro: arXiv:1311.3161.

Troyer and Wiese, PRL, 2008.
 Hastings, J. Math. Phys (2015)

Marvian, Hen, Lidar, arXiv:1802.03408.
 Klassen and Terhal, arXiv:1806.05405.
 Klassen, Marvian *et al.*, arXiv:1906.08800.

What is the optimal basis?

Recall the sample complexity of QMC: $s \sim \langle \text{sign} \rangle_p^{-2} - 1$.

What is the optimal basis?

Recall the sample complexity of QMC: $s \sim \langle \text{sign} \rangle_p^{-2} - 1$.

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[|UT_m U^\dagger|^m - (UT_m U^\dagger)^m \right]$$

What is the optimal basis?

Recall the sample complexity of QMC: $s \sim \langle \text{sign} \rangle_p^{-2} - 1$.

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[|UT_m U^\dagger|^m - (UT_m U^\dagger)^m \right]$$

Computing the average sign via Monte Carlo

$$\frac{\text{Var}[\langle \text{sign} \rangle_q]}{\langle \text{sign} \rangle^2} = \frac{1}{\langle \text{sign} \rangle^2} - 1$$

What is the optimal basis?

Recall the sample complexity of QMC: $s \sim \langle \text{sign} \rangle_p^{-2} - 1$.

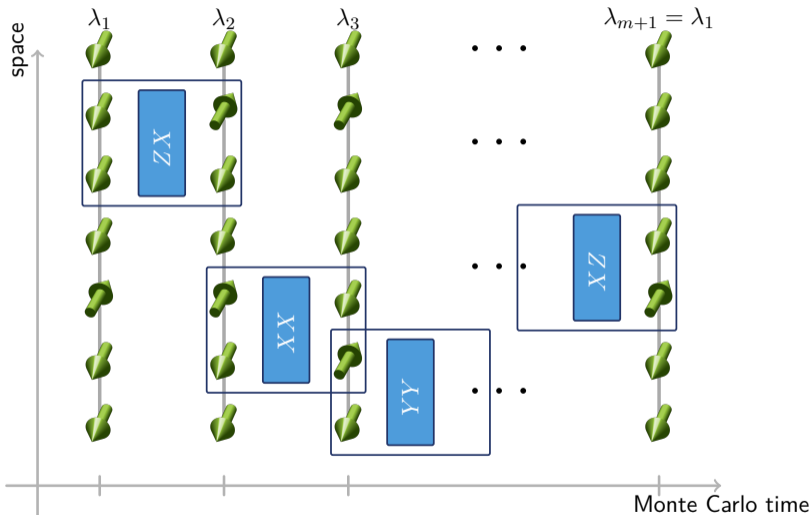
Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[|UT_m U^\dagger|^m - (UT_m U^\dagger)^m \right]$$

Computing the average sign via Monte Carlo

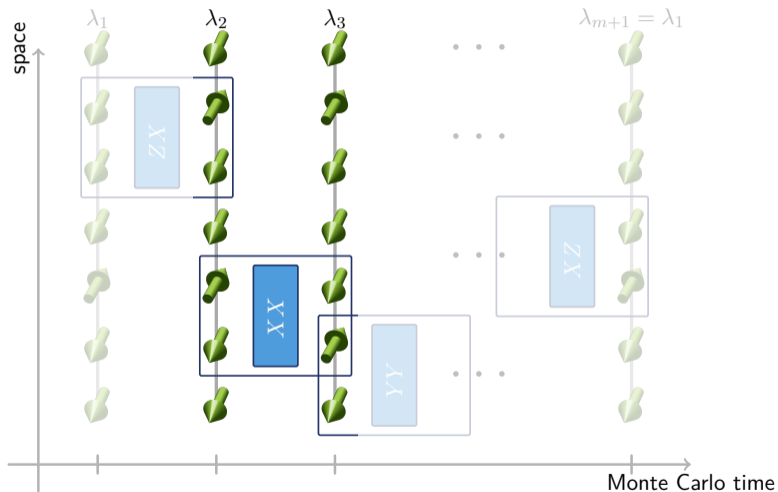
$$\frac{\text{Var}[\langle \text{sign} \rangle_q]}{\langle \text{sign} \rangle^2} = \frac{1}{\langle \text{sign} \rangle^2} - 1$$

What is the optimal basis?



$$a(\lambda) = |T_m(\lambda_1|\lambda_2)| \cdot |T_m(\lambda_2|\lambda_3)| \cdot |T_m(\lambda_3|\lambda_4)| \cdot \cdots \cdot |T_m(\lambda_m|\lambda_1)|$$

What is the optimal basis?



$$a(\lambda) = T_m(\lambda_1|\lambda_2) \cdot T_m(\lambda_2|\lambda_3) \cdot T_m(\lambda_3|\lambda_4) \cdot \dots \cdot T_m(\lambda_m|\lambda_1)$$

Non-stoquasticity

$$\begin{pmatrix} - & + & + & 0 \\ + & + & 0 & - \\ + & 0 & - & 0 \\ 0 & - & 0 & 0 \end{pmatrix}$$

Non-stoquasticity

$$\begin{pmatrix} - & + & + & 0 \\ + & + & 0 & - \\ + & 0 & - & 0 \\ 0 & - & 0 & 0 \end{pmatrix}$$

Non-stoquasticity

$$\left| \begin{array}{cccc} - & + & + & 0 \\ + & + & 0 & - \\ + & 0 & - & 0 \\ 0 & - & 0 & 0 \end{array} \right|_{\ell_1}$$

Easing the sign problem: average sign vs. non-stoquasticity

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[|UT_m U^\dagger|^m - (UT_m U^\dagger)^m \right]$$

Easing the sign problem: average sign vs. non-stoquasticity

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[|UT_m U^\dagger|^m - (UT_m U^\dagger)^m \right]$$

The non-stoquasticity

$$\nu_1(UHU^\dagger) := \| |UT_m U^\dagger| - UT_m U^\dagger \|_{\ell_1} = \| (UHU^\dagger)_{\text{non-stoq.}} \|_{\ell_1}$$

can be *computed efficiently* for 2-local Hamiltonians

Easing the sign problem: average sign vs. non-stoquasticity

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[\underbrace{|\text{UT}_m \text{U}^\dagger|^m - (\text{UT}_m \text{U}^\dagger)^m}_{s(U)} \right]$$

The non-stoquasticity

$$\nu_1(UHU^\dagger) := \|\text{UT}_m \text{U}^\dagger - U \text{UT}_m \text{U}^\dagger\|_{\ell_1} = \|(UHU^\dagger)_{\text{non-stoq.}}\|_{\ell_1}$$

can be *computed efficiently* for 2-local Hamiltonians

Average sign vs. non-stoquasticity: Analytical evidence

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[\underbrace{|UT_m U^\dagger|^m - (UT_m U^\dagger)^m}_{s(U)} \right]$$

Average sign vs. non-stoquasticity: Analytical evidence

Minimize $1/\langle \text{sign} \rangle_p$ over $U \in \mathcal{U}$

$$\arg \min_{U \in \mathcal{U}} \frac{1}{\langle \text{sign} \rangle_p} = \arg \min_{U \in \mathcal{U}} \text{Tr} \left[\underbrace{||UT_m U^\dagger|^m - (UT_m U^\dagger)^m}_{S(U)} \right]$$

To first order in the *non-stoquastic matrix entries*:

$$\begin{aligned} S(U) &\approx 2m \left\| |UT_m U^\dagger| - UT_m U^\dagger \right\|_{\ell_1} \cdot \left\| (|UT_m U^\dagger| + UT_m U^\dagger)^{m-1} \right\|_{\ell_\infty} \\ &\propto d \nu_1(H). \end{aligned}$$

Average sign vs. non-stoquasticity: Analytical evidence

For generic instances, we expect

$$1/\langle \text{sign} \rangle \propto \exp(d\nu_1(H))$$

To first order in the *non-stoquastic matrix entries*:

$$\begin{aligned} S(U) &\approx 2m \left\| |UT_m U^\dagger| - UT_m U^\dagger \right\|_{\ell_1} \cdot \left\| (|UT_m U^\dagger| + UT_m U^\dagger)^{m-1} \right\|_{\ell_\infty} \\ &\propto d\nu_1(H). \end{aligned}$$

Average sign vs. non-stoquasticity: Analytical evidence

For generic instances, we expect

$$1/\langle \text{sign} \rangle \propto \exp(d\nu_1(H))$$

To first order in the *non-stoquastic matrix entries*:

$$\begin{aligned} S(U) &\approx 2m \left\| |UT_m U^\dagger| - UT_m U^\dagger \right\|_{\ell_1} \cdot \left\| (|UT_m U^\dagger| + UT_m U^\dagger)^{m-1} \right\|_{\ell_\infty} \\ &\propto d\nu_1(H). \end{aligned}$$

Average sign vs. non-stoquasticity: Analytical evidence

For generic instances, we expect

$$1/\langle \text{sign} \rangle \propto \exp(d\nu_1(H))$$

SPARSITY

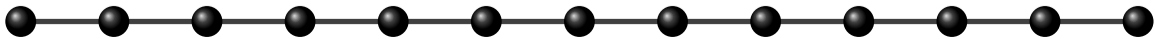
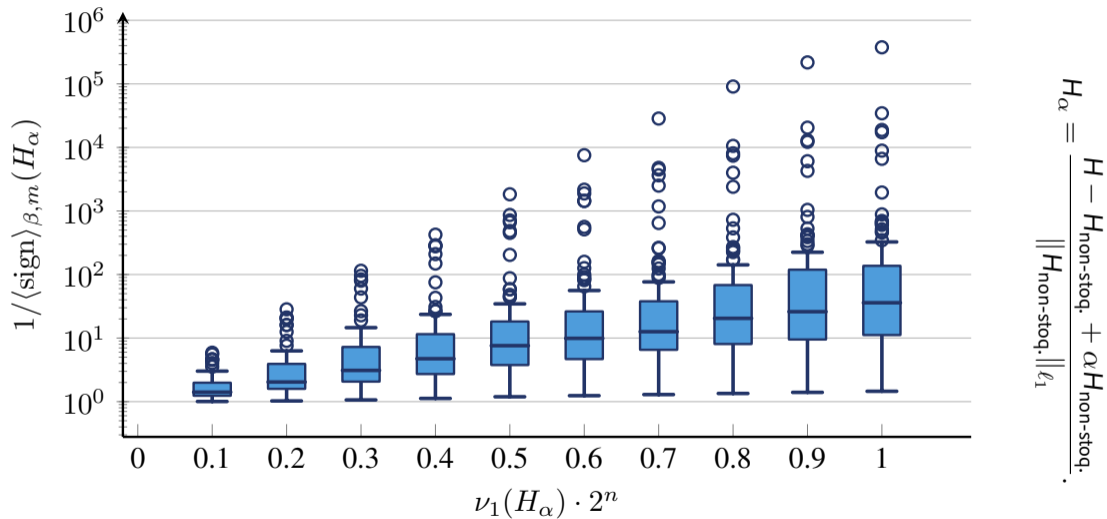
ℓ_1 -norm

NEGATIVITY

To first order in the *non-stoquastic matrix entries*:

$$\begin{aligned} S(U) &\approx 2m \left\| |UT_m U^\dagger| - UT_m U^\dagger \right\|_{\ell_1} \cdot \left\| (|UT_m U^\dagger| + UT_m U^\dagger)^{m-1} \right\|_{\ell_\infty} \\ &\propto d\nu_1(H). \end{aligned}$$

Average sign vs. non-stoquasticity: Numerical evidence



Easing in practice: Translation-invariant problems

Translation-invariant non-stoquasticity

$$H = \sum_{i=1}^n T_i(h)$$

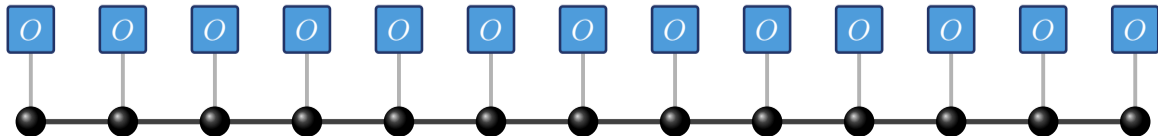
$$\longrightarrow \nu_1(H) \propto \tilde{\nu}_1(h)$$



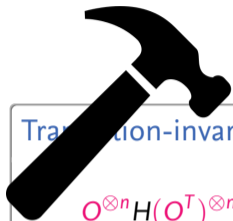
Easing in practice: Translation-invariant problems

Translation-invariant non-stoquasticity

$$\begin{aligned} \mathcal{O}^{\otimes n} H (\mathcal{O}^T)^{\otimes n} &= \sum_{i=1}^n T_i((\mathcal{O} \otimes \mathcal{O}) h(\mathcal{O}^T \otimes \mathcal{O}^T)) \\ \longrightarrow \nu_1(H) &\propto \tilde{\nu}_1((\mathcal{O} \otimes \mathcal{O}) h(\mathcal{O}^T \otimes \mathcal{O}^T)) \end{aligned}$$



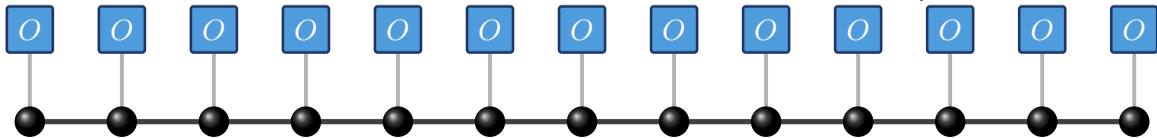
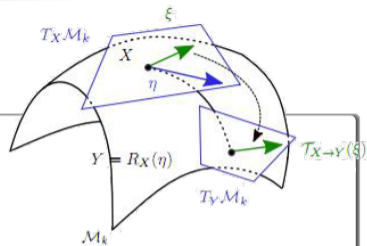
Easing in practice: Translation-invariant problems



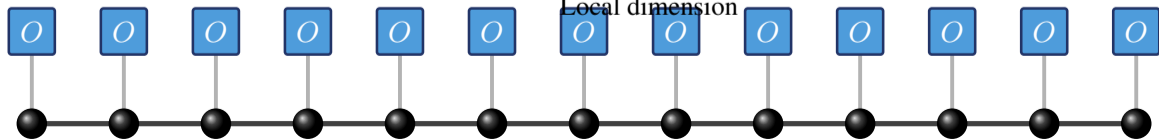
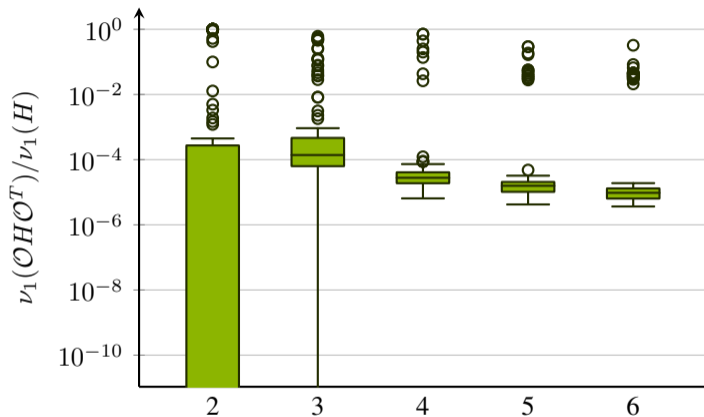
Translation-invariant non-stoquasticity

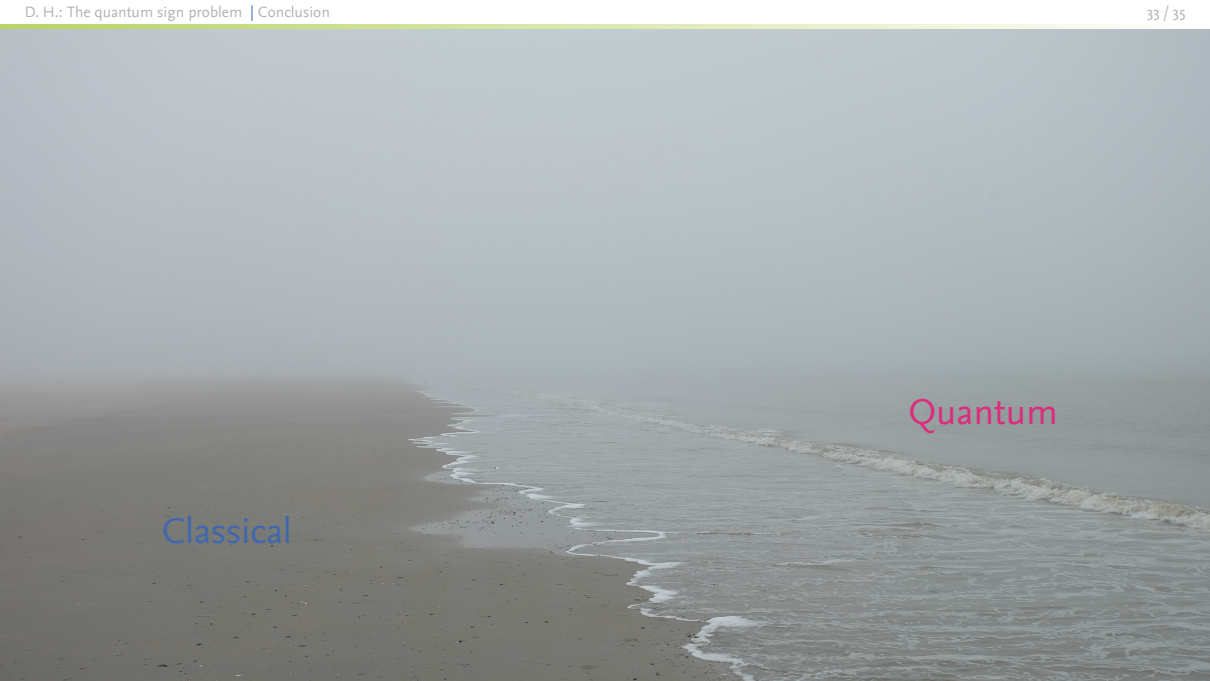
$$\mathcal{O}^{\otimes n} H (\mathcal{O}^T)^{\otimes n} = \sum_{i=1}^n T_i((\mathcal{O} \otimes \mathcal{O}) h(\mathcal{O}^T \otimes \mathcal{O}^T))$$

$$\longrightarrow \nu_1(H) \propto \tilde{\nu}_1((\mathcal{O} \otimes \mathcal{O}) h(\mathcal{O}^T \otimes \mathcal{O}^T))$$



Easing in practice: Translation-invariant problems





Classical

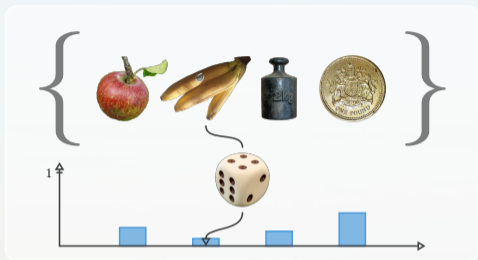
Quantum

Classical

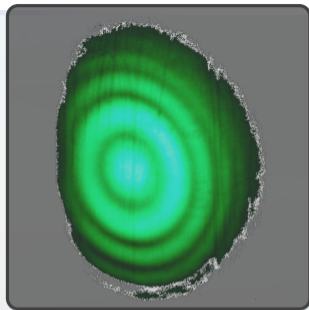
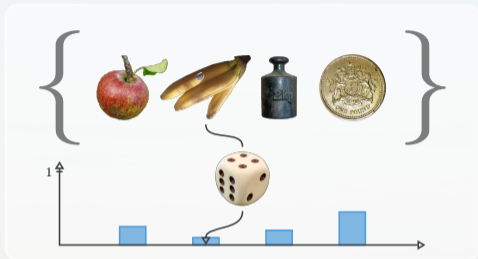
Quantum



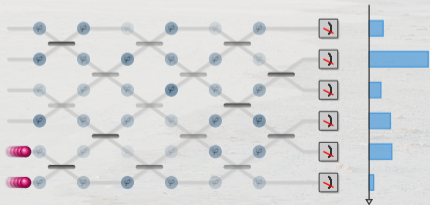
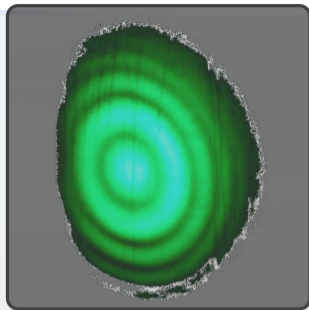
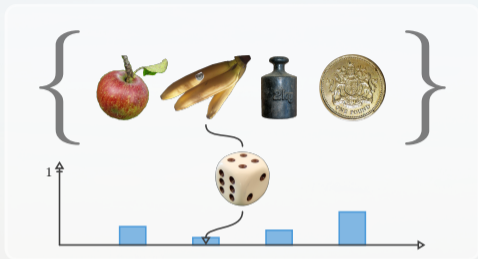
Outlook



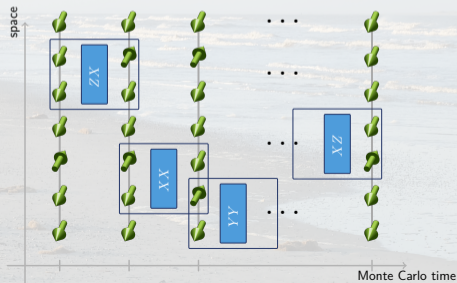
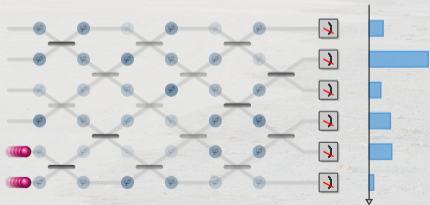
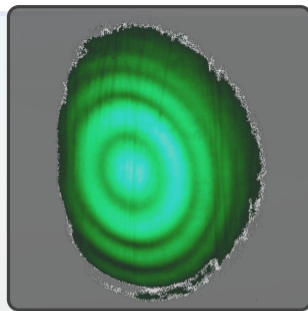
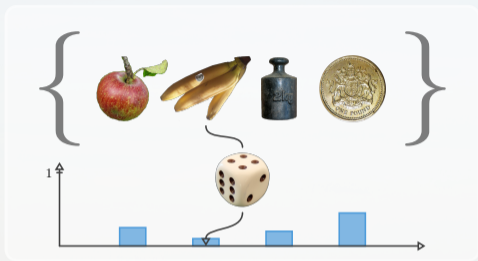
Outlook



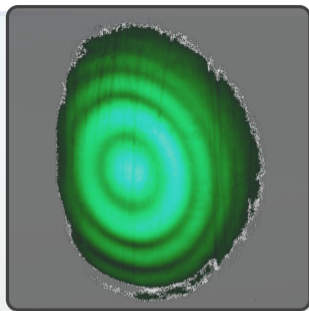
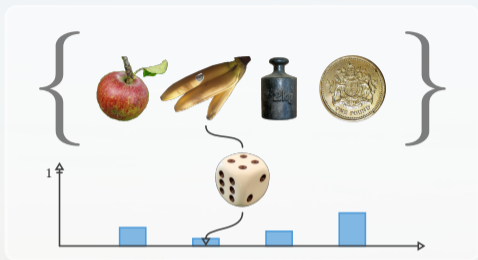
Outlook



Outlook



Outlook



Is the quantum sign problem intrinsic or artificial? [ZOR20]