JOINT CENTER FOR QUANTUM INFORMATION AND COMPUTER SCIENCE

UNIVERSITY OF MARYLAND 1856

Validated quantum advantage via Bell sampling

arXiv:2306.00083

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Simons Workshop, July 10, 2023

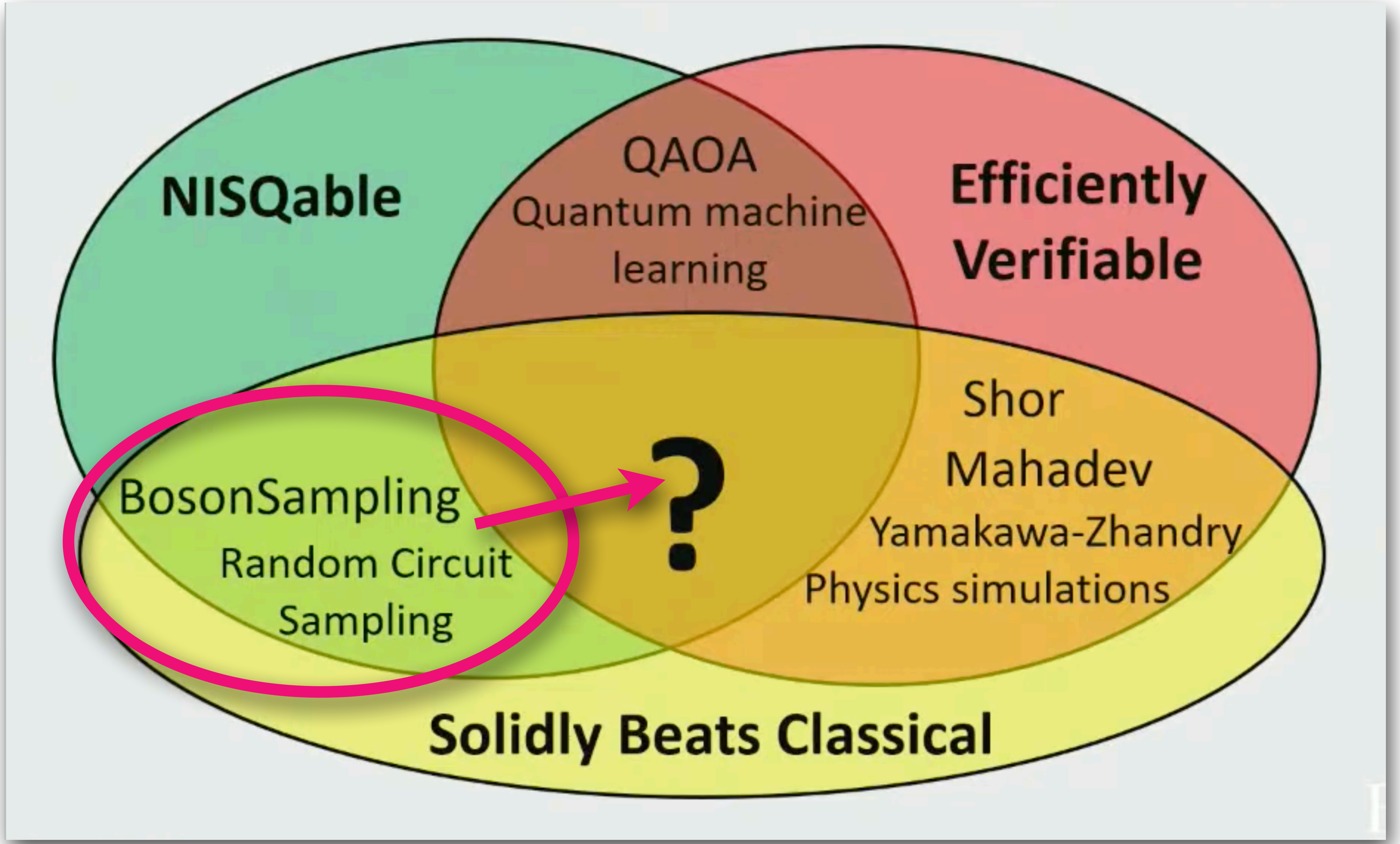
with Michael Gullans

Quantum advantage demonstrations

Is there a

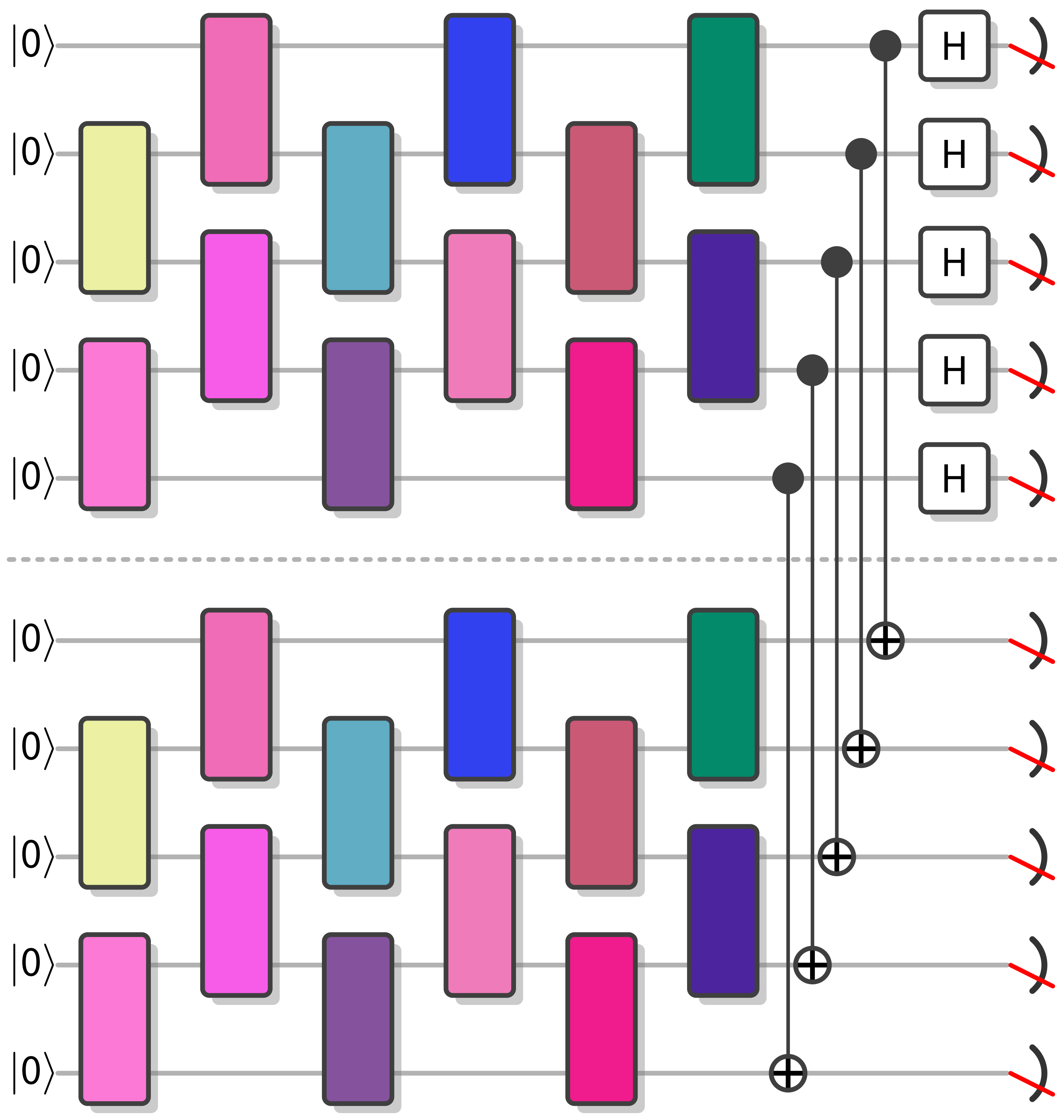
- Scalable
- Verifiable
- NISQ

Quantum advantage?



Aaronson, Simons Workshop 2022

Bell sampling highlights



- → Hardness of sampling
- → Structured distribution with efficient & inefficient property tests
- → **Efficient fidelity estimation** in the same regime in which XEB works

→ Quantum advantage test that is more difficult to spoof than just XEB (**?**)

→ More noise robust than standard-basis sampling (**?**)

Plan for the Talk

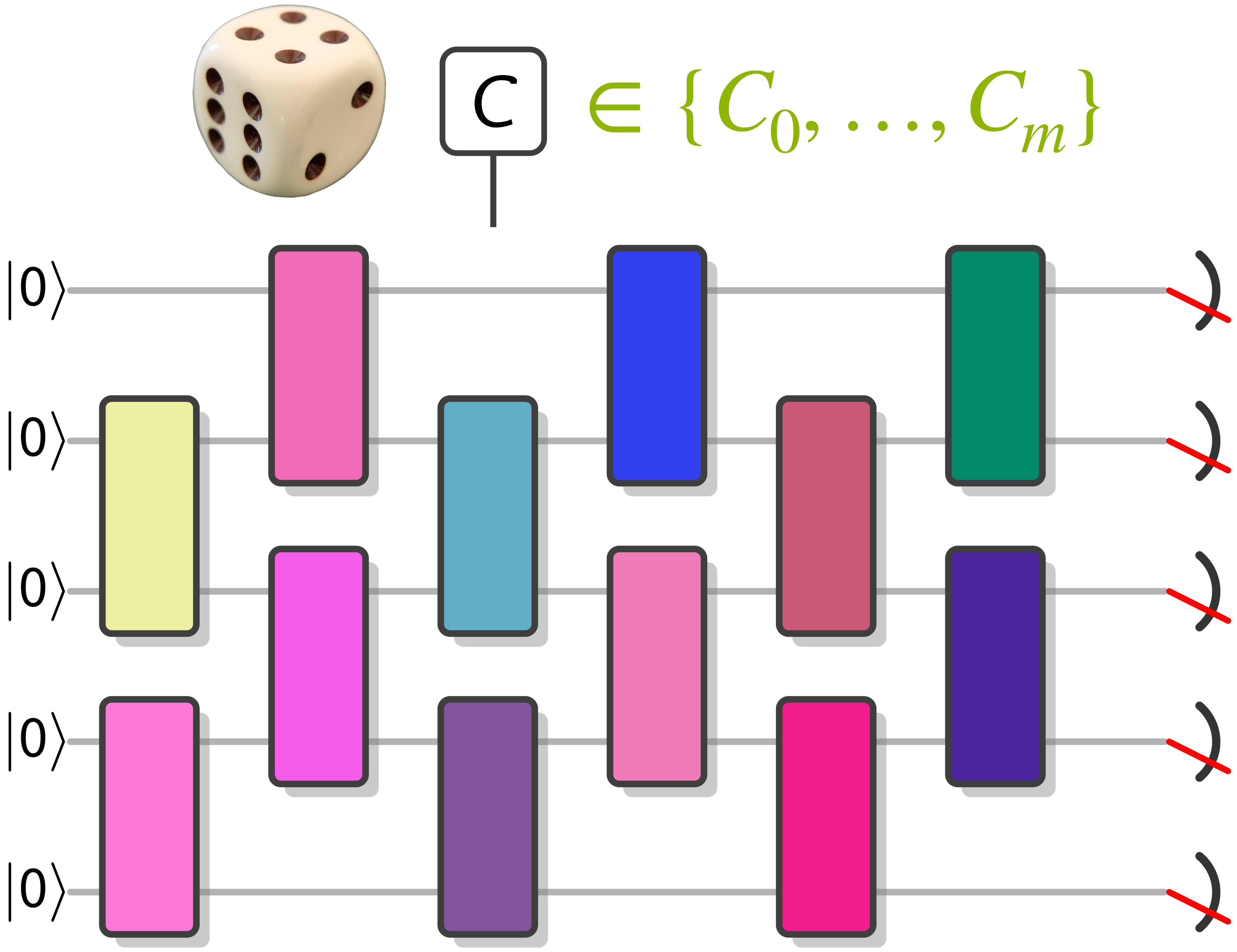
1. Recap of quantum random sampling
2. **Bell sampling**
3. **Noisy Bell sampling**
4. Bell sampling in experiments
5. Scalability
6. Outlook & open questions

Do you have questions?

Quantum random sampling

---> a brief recap of the noiseless case

Quantum random sampling



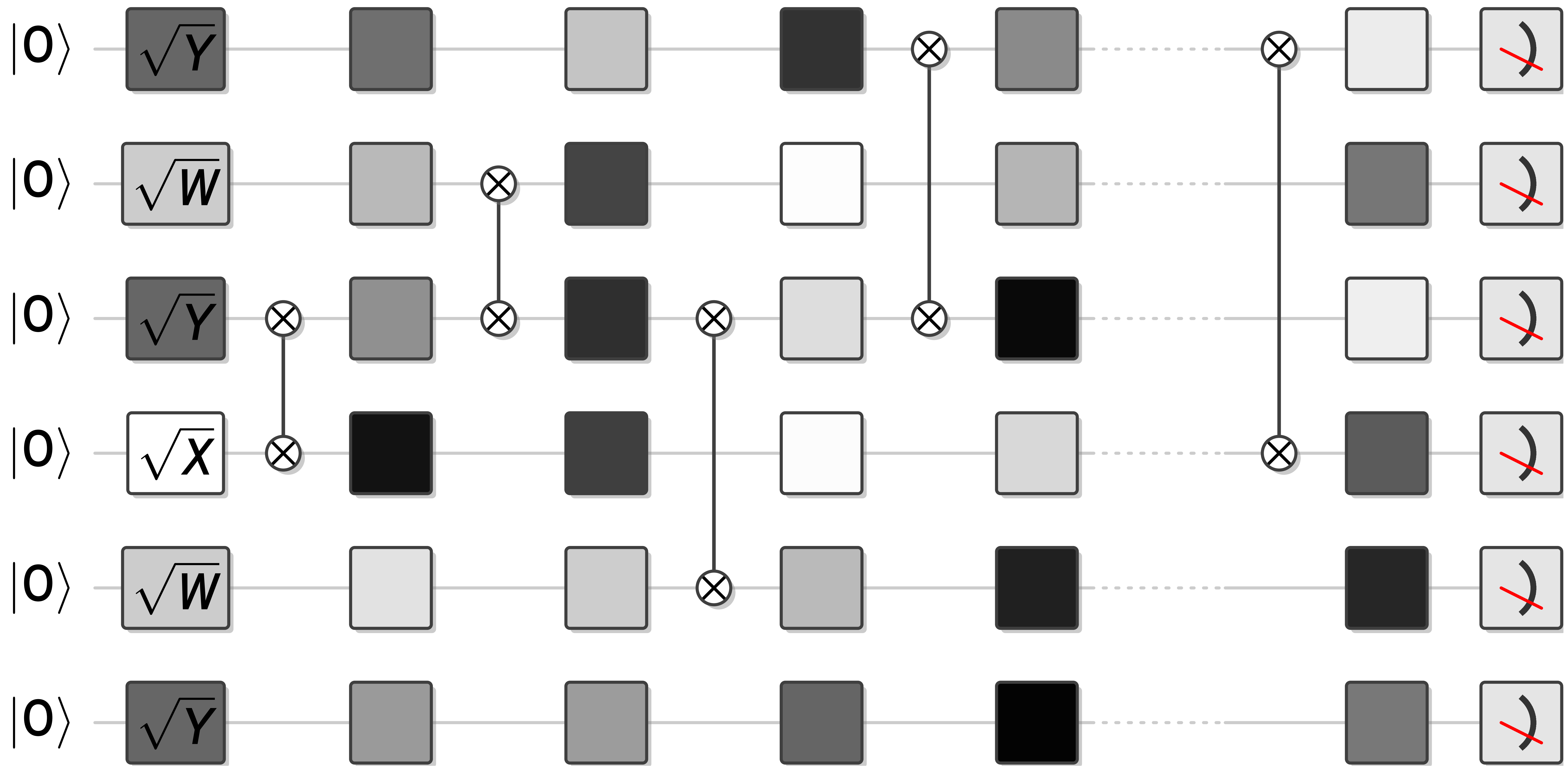
$\in \{C_0, \ldots, C_m\}$

TASK

1. Choose a random circuit C.
2. Sample from the output distribution p_C.

$$S \leftarrow P_C(S) = |\langle S | C | 0 \rangle|^2$$

For example: The Google circuits



Evidence for hardness

Assuming some complexity-theoretic conjectures, there is no efficient classical algorithm $\mathscr{A}$ that approximately samples from p_C in total-variation distance.

Aaronson & Arkhipov, 2013; Bremner, Jozsa, Shepherd, 2010; Bremner, Montanaro, Shepherd, 2016

Conjecture (XHOG): Producing n outcomes $x_1, \ldots, x_n$ with average probability $\mathbb{E}_i[P_C(x_i)] \geq b/2^n$ is classically intractable.

Aaronson & Chen, 2017; Aaronson & Gunn, 2020

The outcome distribution

Verification & benchmarking of QRS

The largest probability of P_C is very small, i.e., $O(1/\sqrt{2^n})$.

–> Verifying total-variation distance requires $\Omega(2^{n/4})$ samples.

Valiant & Valiant, 2015; D.H. *et al.*, 2018

U forms a 2-design

Cross-entropy benchmarking (XEB)

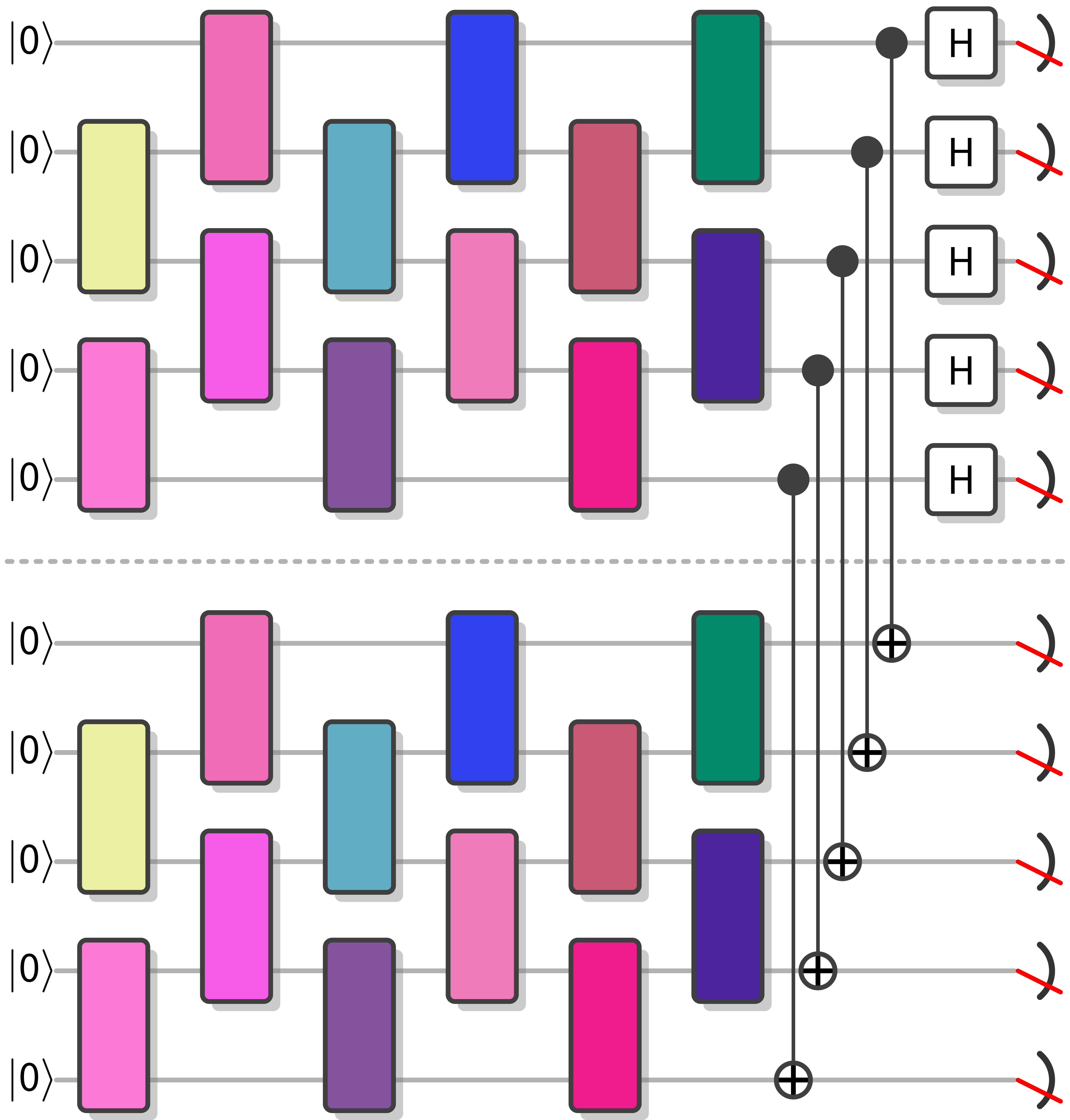
$$\overline{F_{\mathrm{XEB},f}}(Q, P_U) = \mathbb{E}_U\left[2^N \sum_{x\in\{0,1\}^n} Q(x)P_U(x) - 1\right] = \begin{cases} 1 & Q = P_U \\ 2^{-n} & Q = 2^{-n} \end{cases}$$

Statistical properties
→ Can be estimated from a polynomial number of samples.
→ Distinguishes from the uniform distribution.

Complexity properties
→ Hard to achieve a high score classically(?)

Bell sampling

Bell sampling



TASK

1. Choose a random circuit C.

$$C \leftarrow \{C_0, \ldots, C_m\}$$

2. Sample from the output distribution P_C.

$$r \leftarrow P_C(r) = |\langle \sigma_r | C \otimes C | 0 \rangle|^2$$

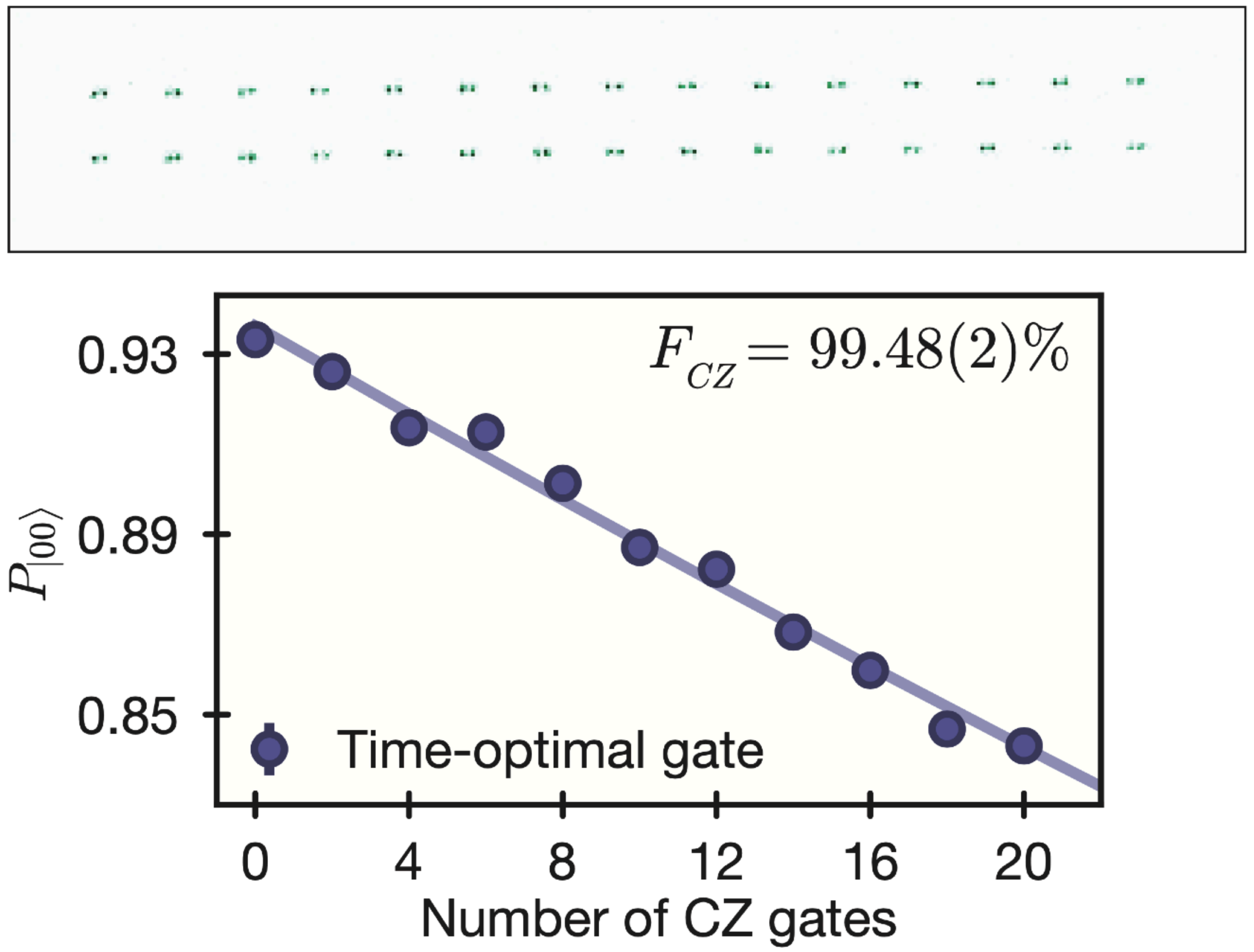
$$r \in \{0,1\}^{2n}$$

$$|\sigma_r\rangle = (\sigma_r \otimes \mathbb{1})(|00\rangle + |11\rangle)/\sqrt{2}$$

$$\sigma_{00} = 1, \sigma_{01} = Z, \sigma_{10} = X, \sigma_{11} = Y$$

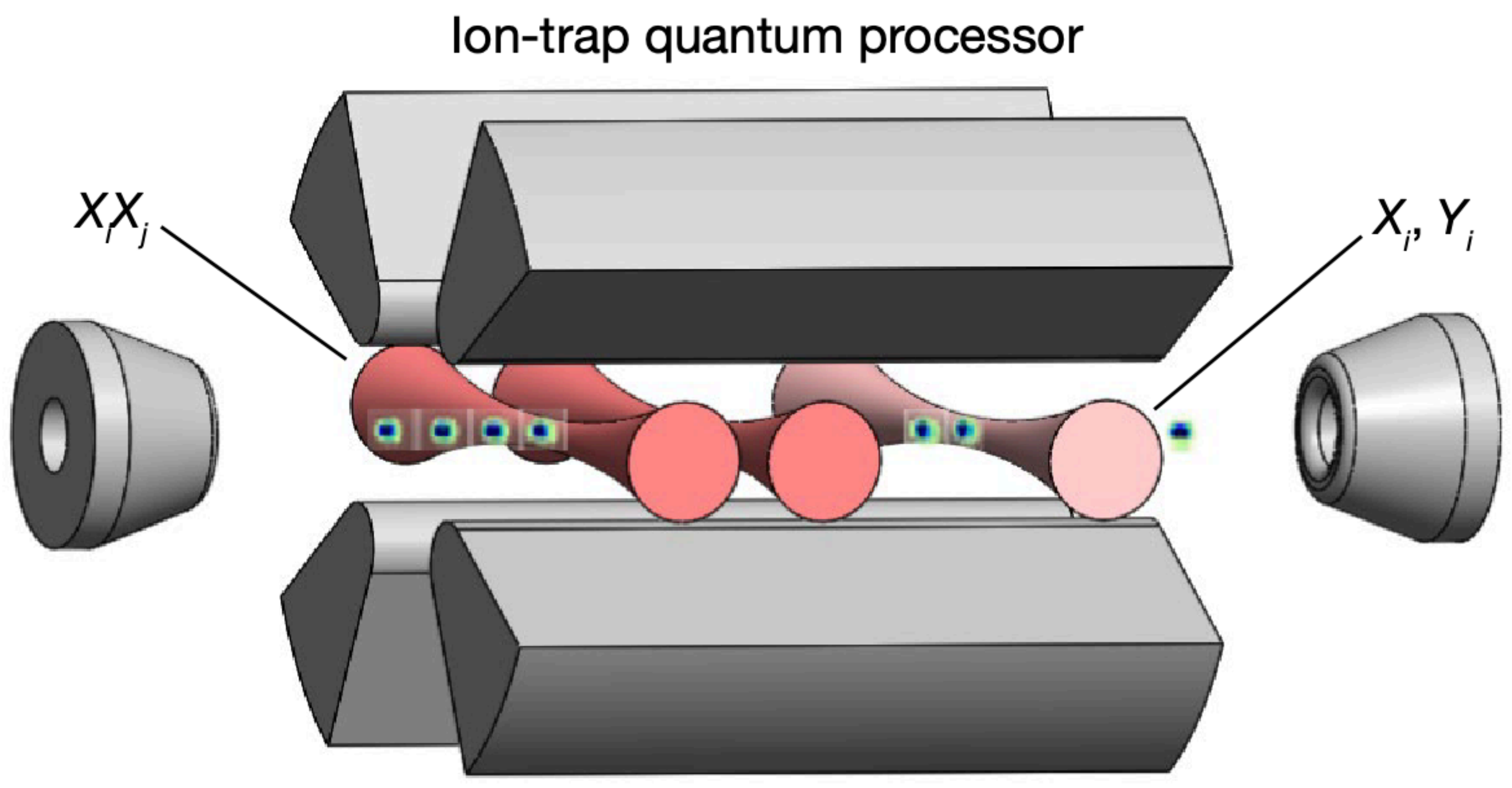
Bell sampling is realistic

Rydberg atoms in tweezers



Evered *et al.*, 2023

Ion traps



Postler *et al.*, 2023

→ Bell measurement just requires one circuit layer!

Bell sampling is hard: the Stockmeyer argument

Hiding

$$P_C(r) = P_{C_s}(r \oplus s) \quad C_s = C \cdot \sigma_s^{1/2}$$

Approximate average-case hardness of computing probabilities

i. Worst-case approximate hardness
ii. Average-case (near) exact hardness AA13; BFNV19
iii. Anticoncentration BMS16, BHH16, HBVSE18, HM23, …

Last layer invariant under $\sigma_s^{1/2}$

Universal quantum circuits

Continuous gate set

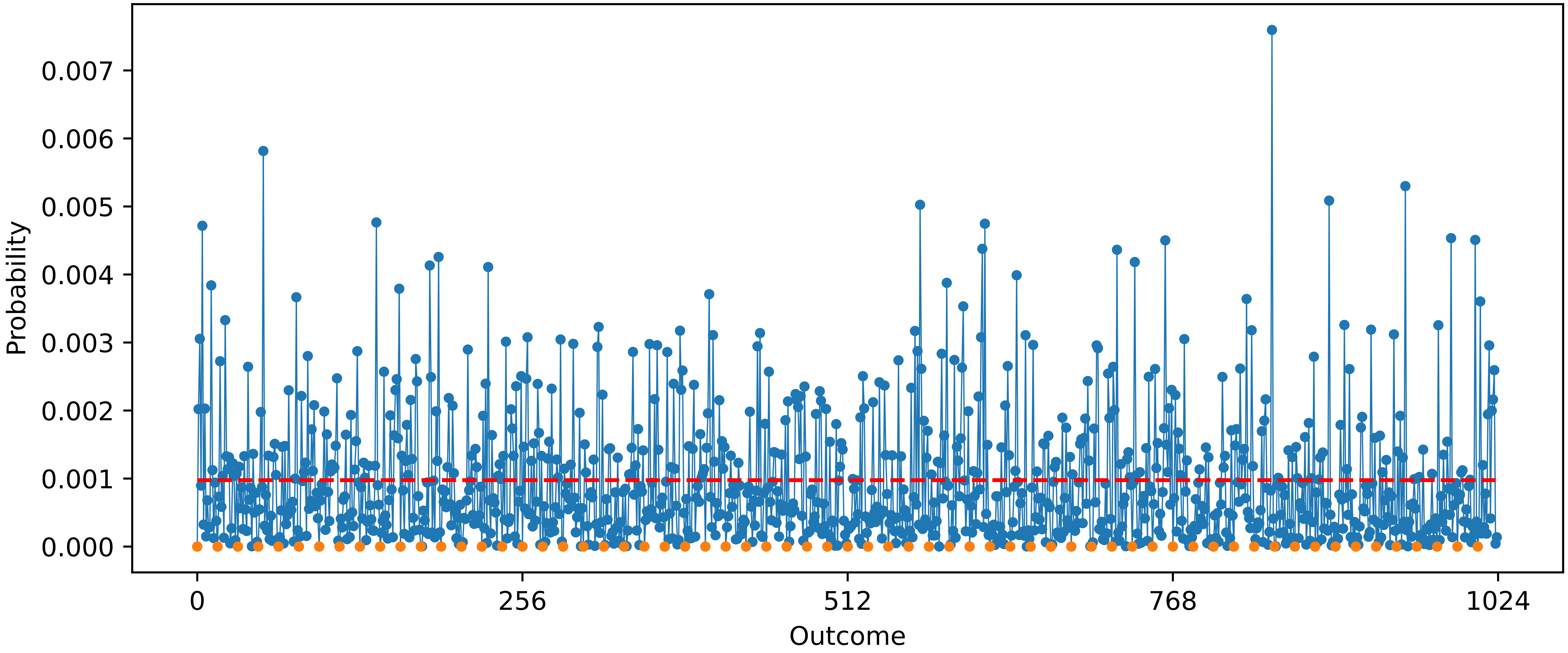
depth $n^{1/D}$

Unitary 4-designs Harrow, Mehraban, 2023

The Bell sampling distribution has structure!

$$P_C(r) = |\langle \sigma_r | C \otimes C | 0 \rangle|^2$$

- Supported on symmetric subspace
- Outcomes are correlated
- Large stabilizer dimension
- Anticoncentrated on the symmetric subspace



Support & the SWAP test

$$\mathbb{S}_2 = |\sigma_{00}\rangle\langle\sigma_{00}| + |\sigma_{10}\rangle\langle\sigma_{10}| + |\sigma_{01}\rangle\langle\sigma_{01}| - |\sigma_{11}\rangle\langle\sigma_{11}|$$

$$\mathbb{S} = \mathbb{S}_2^{\otimes n}$$

→ $\mathrm{Tr}[\rho^2] = \mathrm{Tr}[(\rho \otimes \rho)\mathbb{S}] = \sum_r (-1)^{\pi_Y(r)} |\langle\sigma_r| C \otimes C |0\rangle|^2$

$\pi_Y(r) =$ parity of 11 outcomes

→ **For** $\rho = |C\rangle\langle C|$: $\quad |\{r : r \leftarrow P_C \wedge \pi_Y(r) = 1\}| = 0$

→ **The distribution is supported on the symmetric subspace ($\pi_Y(r) = 0$)**

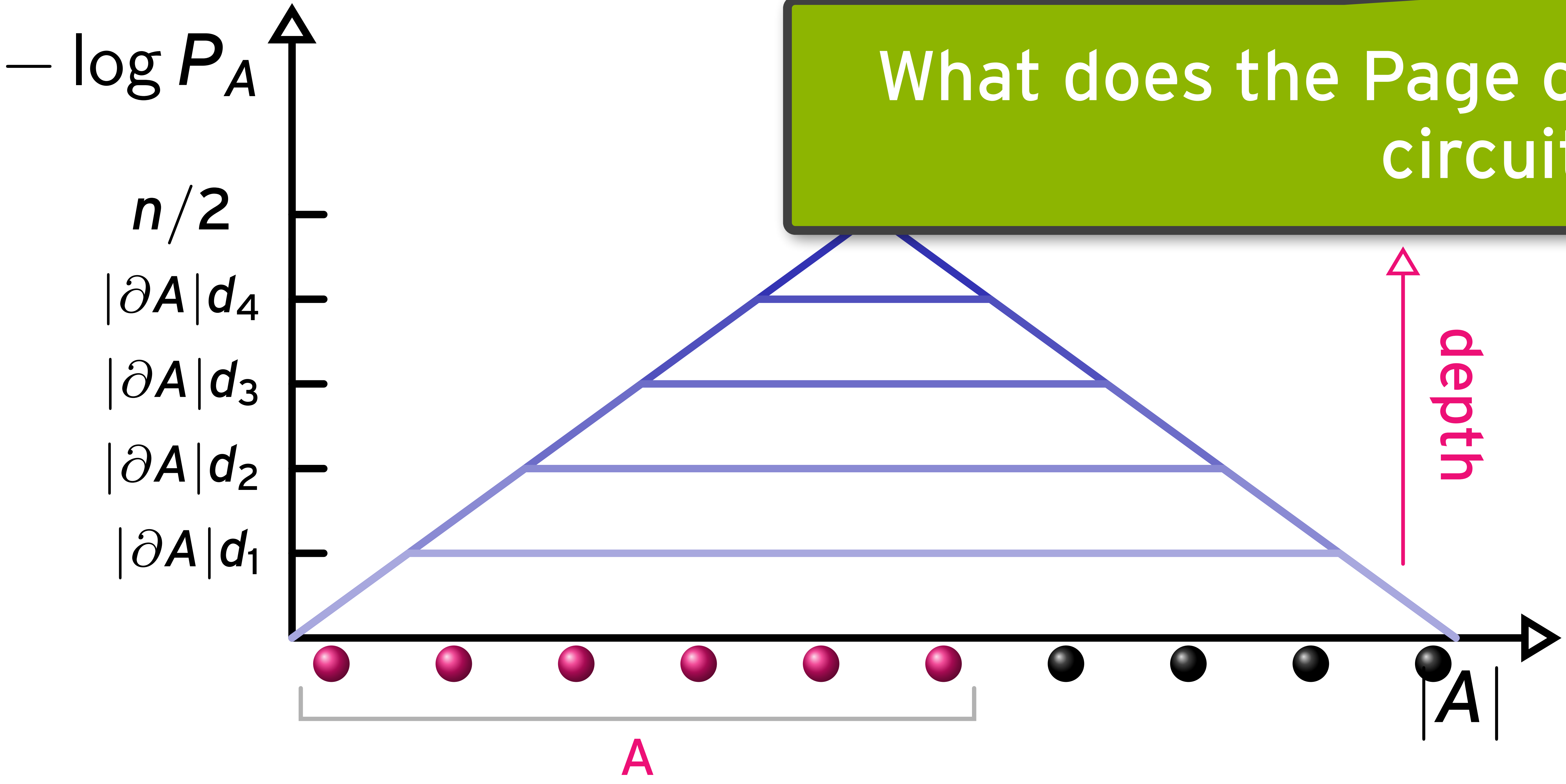
→ **Can be efficiently checked: SWAP test**

$$\mathrm{Tr}[\rho^2] \approx \frac{1}{M}(|\{r : \pi_Y(r) = 0\}| - |\{r : \pi_Y(r) = 1\}|)$$

Correlations & the subsystem SWAP test

- Subsystem purities reflect the entanglement properties.

$$P_A = \mathrm{Tr}[\rho_A^2] \approx \frac{1}{M}\left(|\{r : \pi_Y(r_A) = 0\}| - |\{r : \pi_Y(r_A) = 1\}|\right)$$



What does the Page curve look like for interesting circuit ensembles?

→ Subsystem SWAP test

- Average entanglement properties can be computed. → **Page curves**

Page, 1992

Stabilizer dimension & magic test

$$P_C(r) = |\langle \sigma_r | C \otimes C | 0 \rangle|^2 = \frac{1}{2^n} |\langle \overline{C} | \sigma_r | C \rangle|^2$$

- For a stabilizer state $|S\rangle$, the distribution is supported (essentially) on the stabilizer of $|S\rangle$.

Montanaro, 2017; Gross, Nezami, Walter, 2022

- If C contains few (t) T gates, then $|C\rangle\langle\overline{C}| = \sum_{\sigma_l \in \mathscr{L}} \lambda_l \sigma_l \Pi_{\mathscr{C}} \sigma_k$
- Stabilizer dimension $\dim(\mathscr{L}) \leq 2t$
- **Magic test:** $\dim(\mathscr{L})$ can be estimated from $O(n \log n/\epsilon)$ Bell samples.

In fact, a Clifford + T circuit with t T-gates can be learned from $O(n \log n/\epsilon)$ Bell samples and $O(2^t/\epsilon^2)$ additional measurements.

See also Grewal, Iyer, Kretschmer, Liang 2023; Leone, Oliveira, Hamma, 2023

The Bell sampling distribution has structure!

$$P_C(r) = \frac{1}{2^n} |\langle \overline{C} | \sigma_r | C \rangle|^2$$

Distribution property		Test	
→ Supported on symmetric subspace	←	Global SWAP test	*efficient*
→ Large stabilizer dimension	←	Magic test	
→ Outcomes are correlated	←	Subsystem SWAP test	
→ Heavy outcomes	←	XEB test	*Inefficient*

Structure gives state distinguishers

$$P_C(r) = \frac{1}{2^n} |\langle \overline{C} | \sigma_r | C \rangle|^2$$

Distinguishes against		Test	
→ Globally uniform	←	Global SWAP test	*efficient*
→ Output distributions of circuits with no ($< \dim(\mathscr{L})$) non-Clifford gates	←	Magic test	
→ Uniform on symmetric subspace	←	Subsystem SWAP test	
→ Wrong heavy outcomes	←	XEB test	*Inefficient*

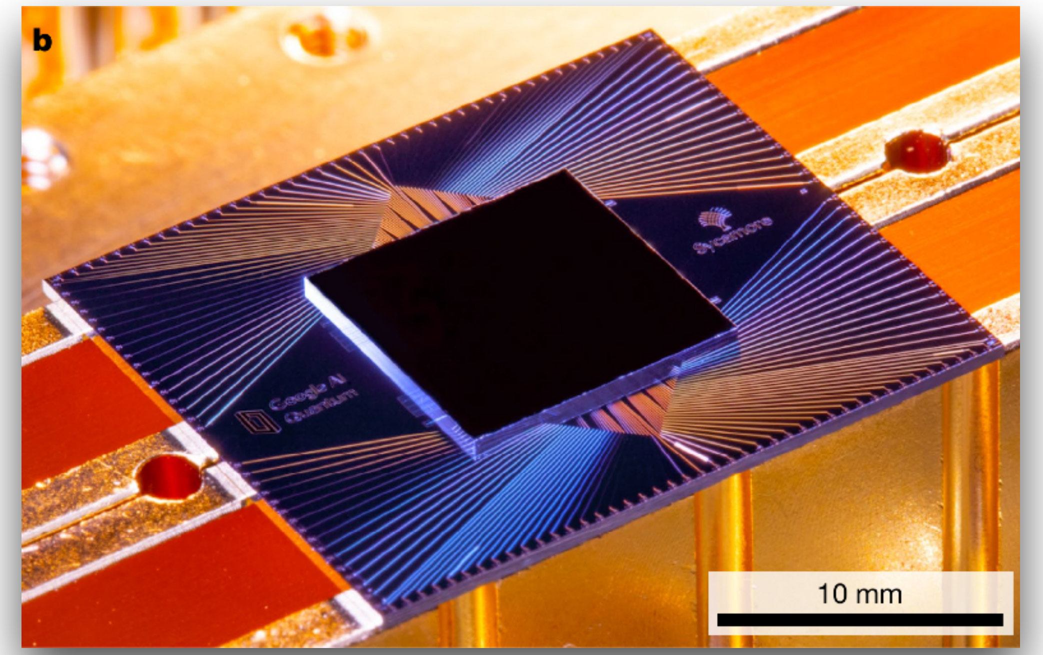
Noisy Bell sampling

Quantum random sampling

---> a brief recap of the noisy case

Noisy quantum random sampling

Noisy quantum device



vs.

Classical algorithm

Exponentially decaying fidelity ← → Exponential computing cost

$\epsilon \in O(1/n)$

XEB

→ Noisy quantum device XEB score scales as $2^{-\Theta(\epsilon \cdot n \cdot d)} = 2^{-\Theta(d)}$.

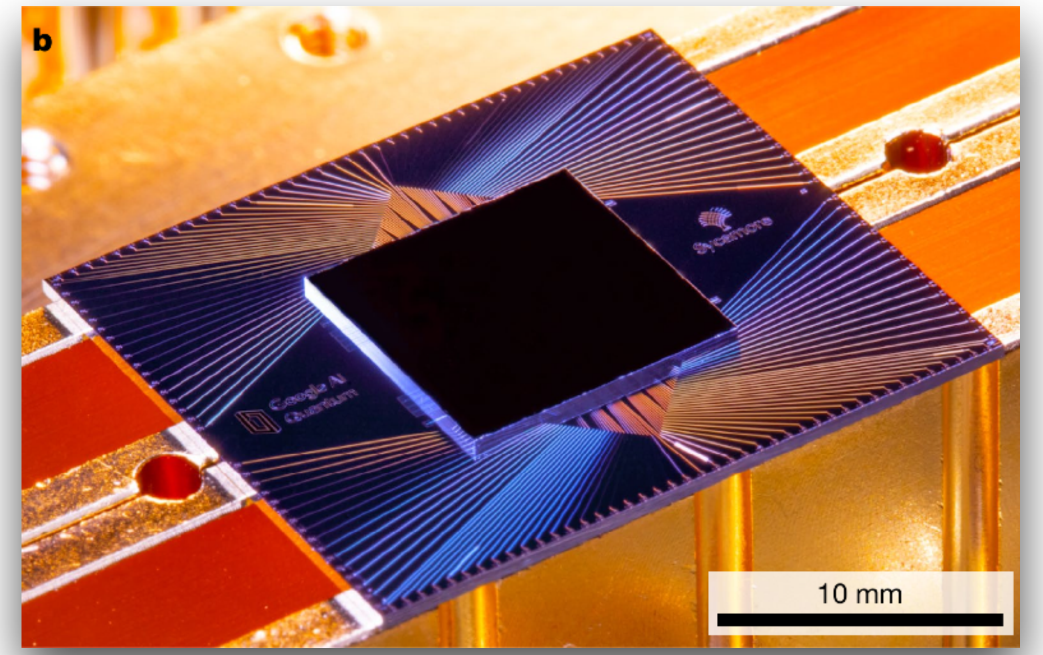
→ XEB score relates to fidelity.

Gao et al, 2021; Morvan et al, 2023; Ware et al. 2023

→ Efficient classical algorithms can achieve score $2^{-O(d)}$ on XEB.

Gao *et al.*, 2021

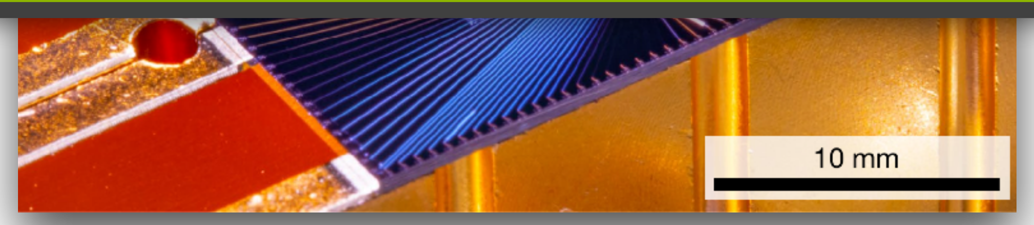
!!! Abhinav's talk !!!



How well can a noisy quantum device/ an efficient classical algorithm do on the Bell sampling tests?

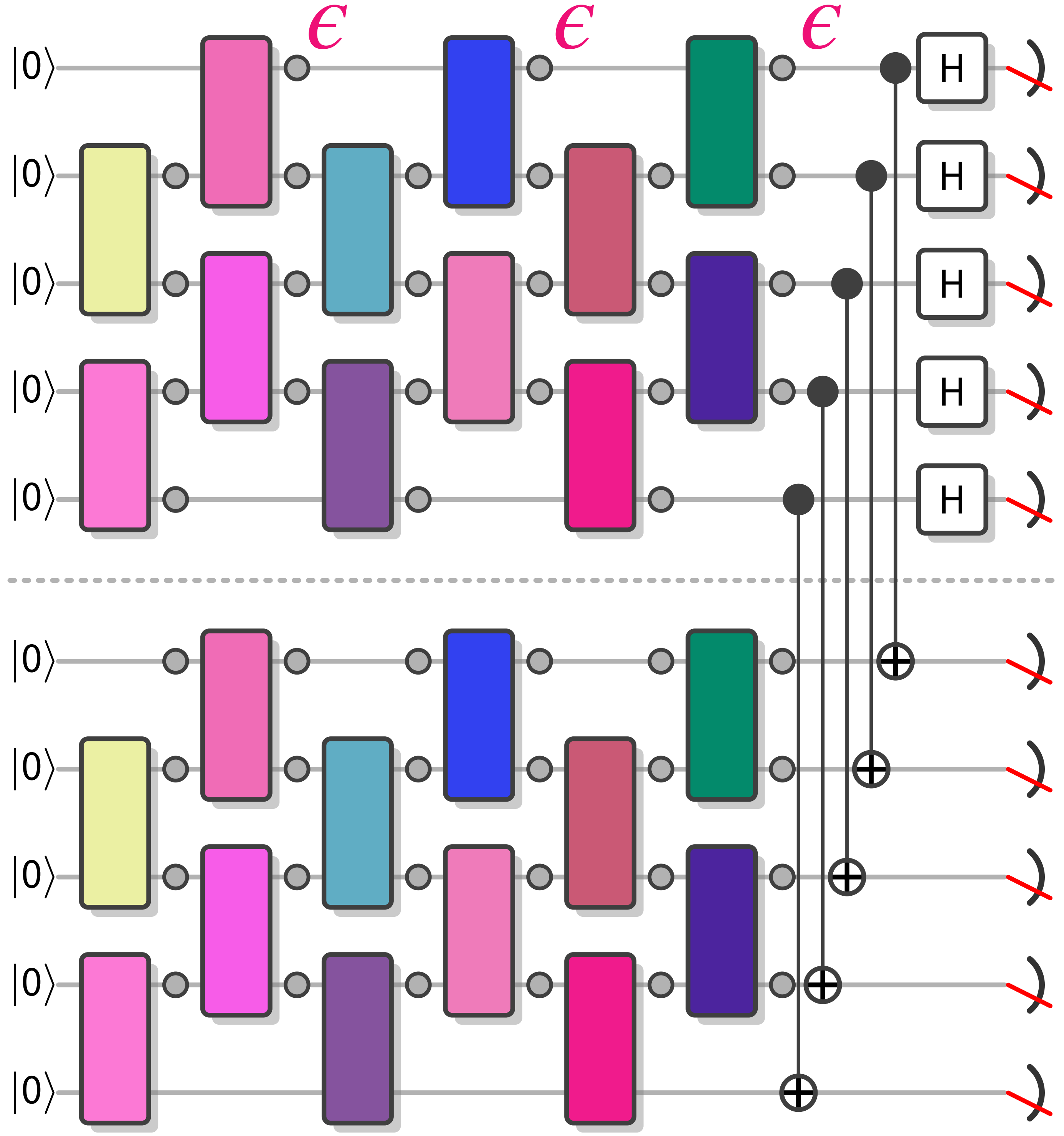
Noisy Bell sampling

Which exactly?
How does it decay in different noise regimes?



→ XEB has similar properties.

→ Consider the SWAP tests.



Make noise assumptions.

→ White-noise approximation on C
Boixo *et al.*, 2018; Dalzell *et al.*, 2021

$\epsilon < 1/n$

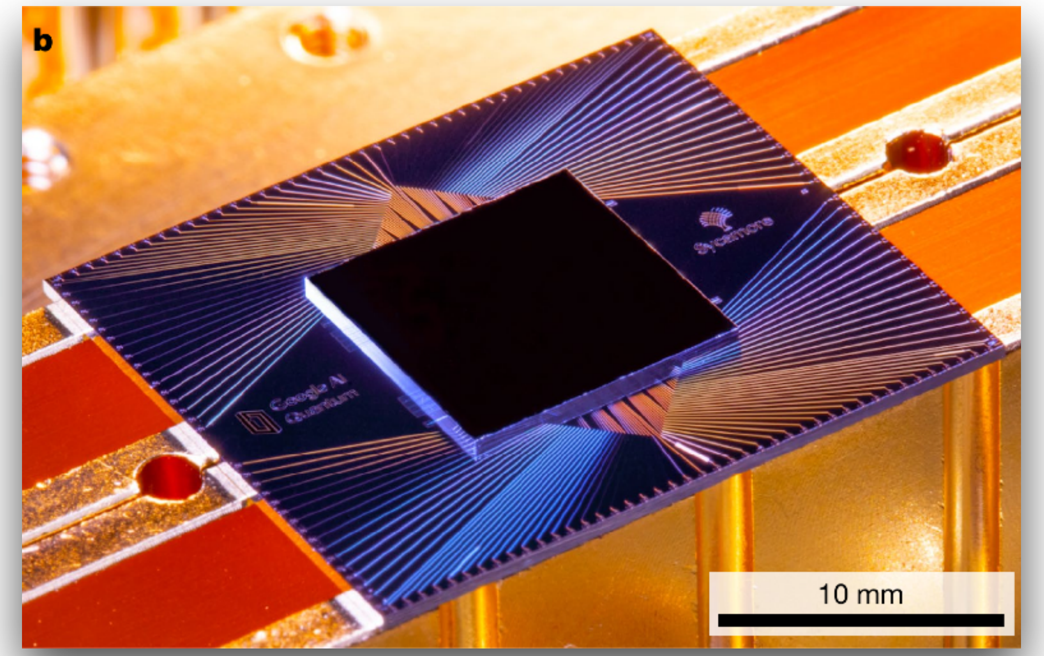
$$\rho_C(\eta) = (1-\eta)|C\rangle\langle C| + \eta \frac{\mathbb{1}}{2^n}$$

→ Ideal measurement

e.g. $A = [n]$ (purity)

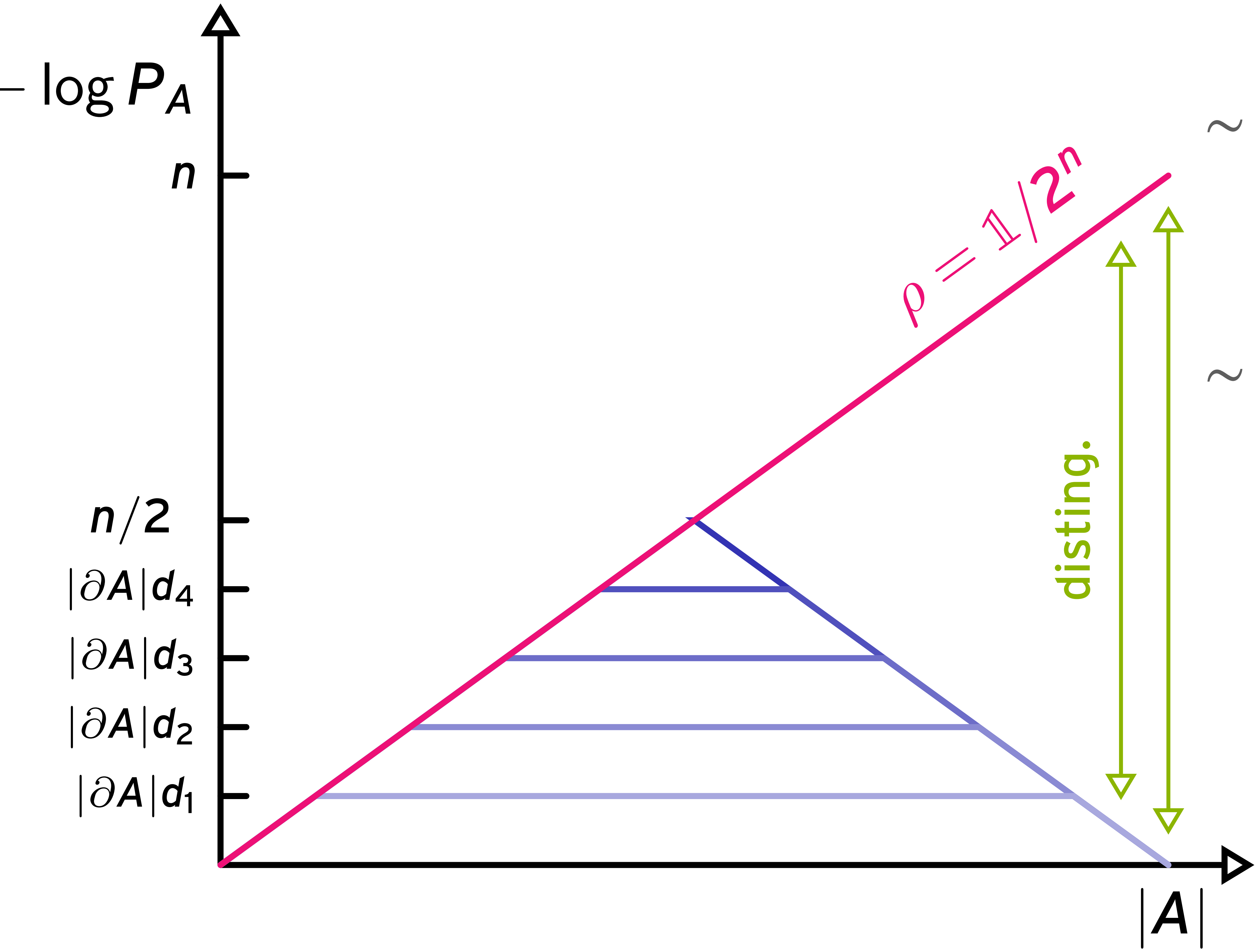
$$P_{[n]}(\rho_C) = 1 - O(\eta)$$

Error detection



→ Every shot can detect errors $\xrightarrow{r \leftarrow Q_C}$ Declare an error

- $\sim \frac{1}{2}$ ☆ $\pi_Y(r) = 1$.
- $\sim \frac{1}{2} - \frac{1}{2^{n-1}}$ ☆ The fraction of subsystems A of size $n-1$ with $\pi_Y(r_A) = 1$ is too large.
- $\sim \frac{1}{2} - \frac{1}{2^{n-2}}$ ☆ The fraction of subsystems A of size $n-2$ with $\pi_Y(r_A) = 1$ is too large.
- ☆ ....



→ White-noise approximation on C

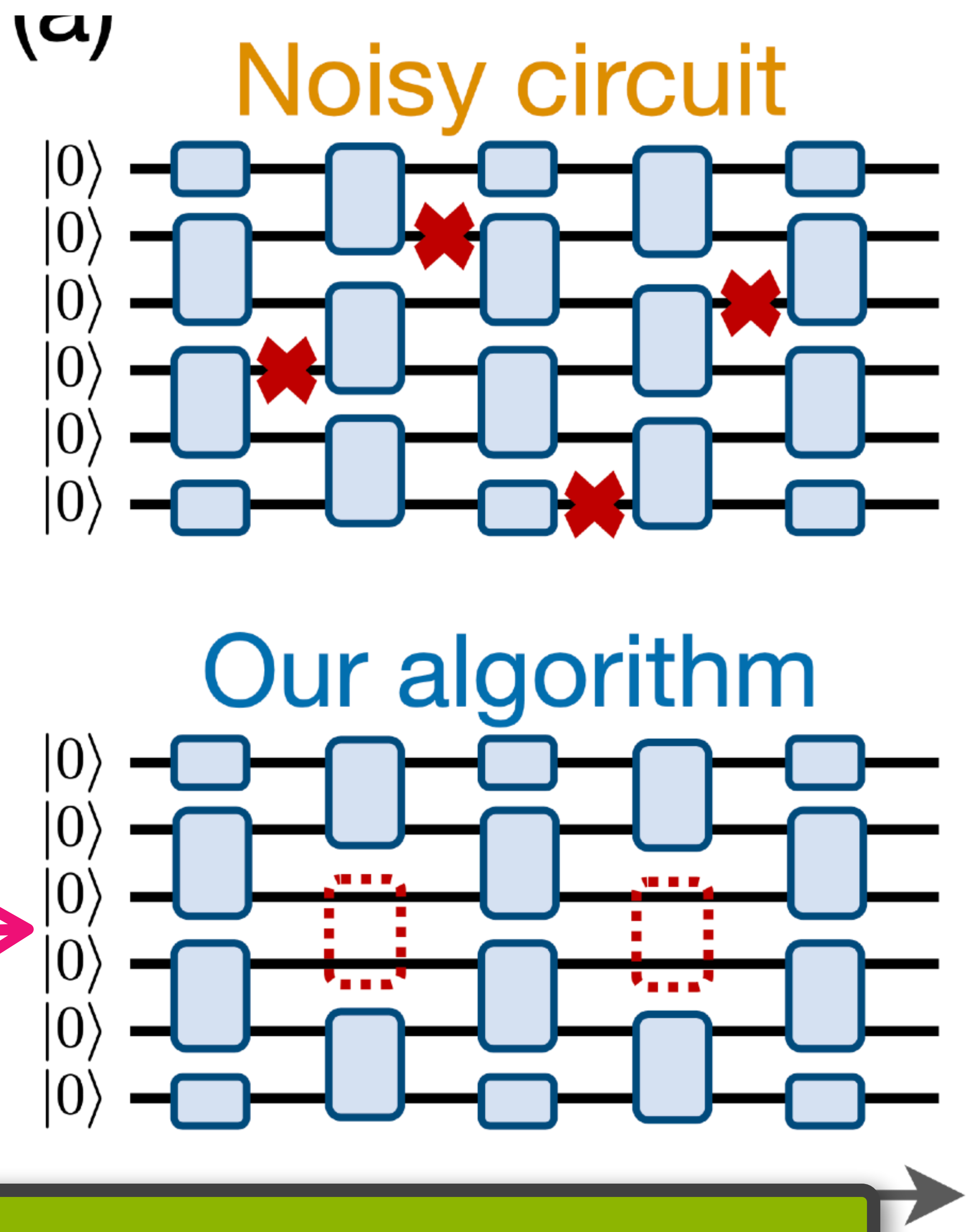
$$\rho_C(\eta) = (1-\eta)|C\rangle\langle C| + \eta \frac{\mathbb{1}}{2^n}$$

→ Ideal measurement

Classical algorithms

- Magic test: uniform samples gives high score!
- Subsystem SWAP test: Clifford circuits generate high entanglement
- Gao *et al.* (2021) spoofer for the XEB still works (?)
  - Can be detected by the SWAP test!

e.g. $\max_{A\subset[n]} P_A(\rho_C)$ or $\max_{A\subset[n]} \left| -\log \left[\overline{P_A} / P_A(\rho_C) \right] \right|$



- Classical algorithms can also do error detection.

How does error detection fare if the distribution does not “contain” the ideal signal?

Proposed quantum advantage test

Do the new tests "add" to XEB?

- High score on XEB: $\overline{2^n \frac{1}{M} \sum_{r \leftarrow Q} P_C(r) - 1}$ *inefficient*
- Good score on SWAP test, e.g.: $\max_{A \subset [n]} \left| -\log \left[\overline{P_A} / P_A(\rho_C) \right] \right|$ *efficient* *for very large/small A*
- High score on magic test: $\dim(\mathscr{L})$ *efficient*

Proposition

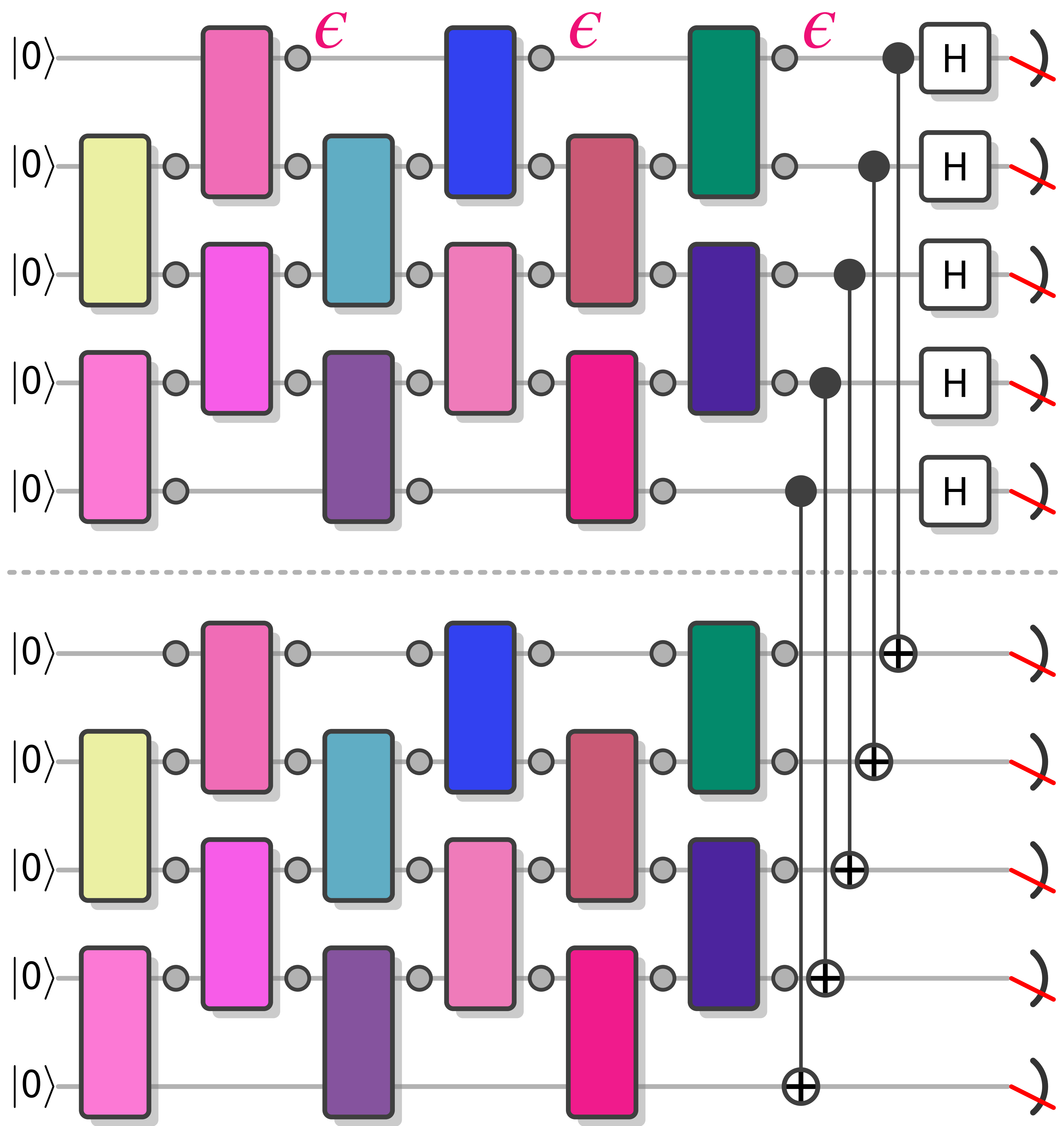
Simultaneously achieving a "good score" on all tests is classically intractable.

Bell sampling in experiments

Measuring fidelity in experiments

→ XEB is an **inefficient** estimator of fidelity in the white-noise regime

!!! *Abhinav's talk* !!!



→ Purity is an **efficient** estimator of fidelity in the white-noise regime.

$$\mathrm{Tr}[\rho_C(\eta)^2] = (1-\eta)^2 + O(1/2^n)$$

$$\langle C|\rho_C(\eta)|C\rangle = 1 - \eta + O(1/2^n)$$

→ White-noise approximation on C

Dalzell *et al.*, 2021

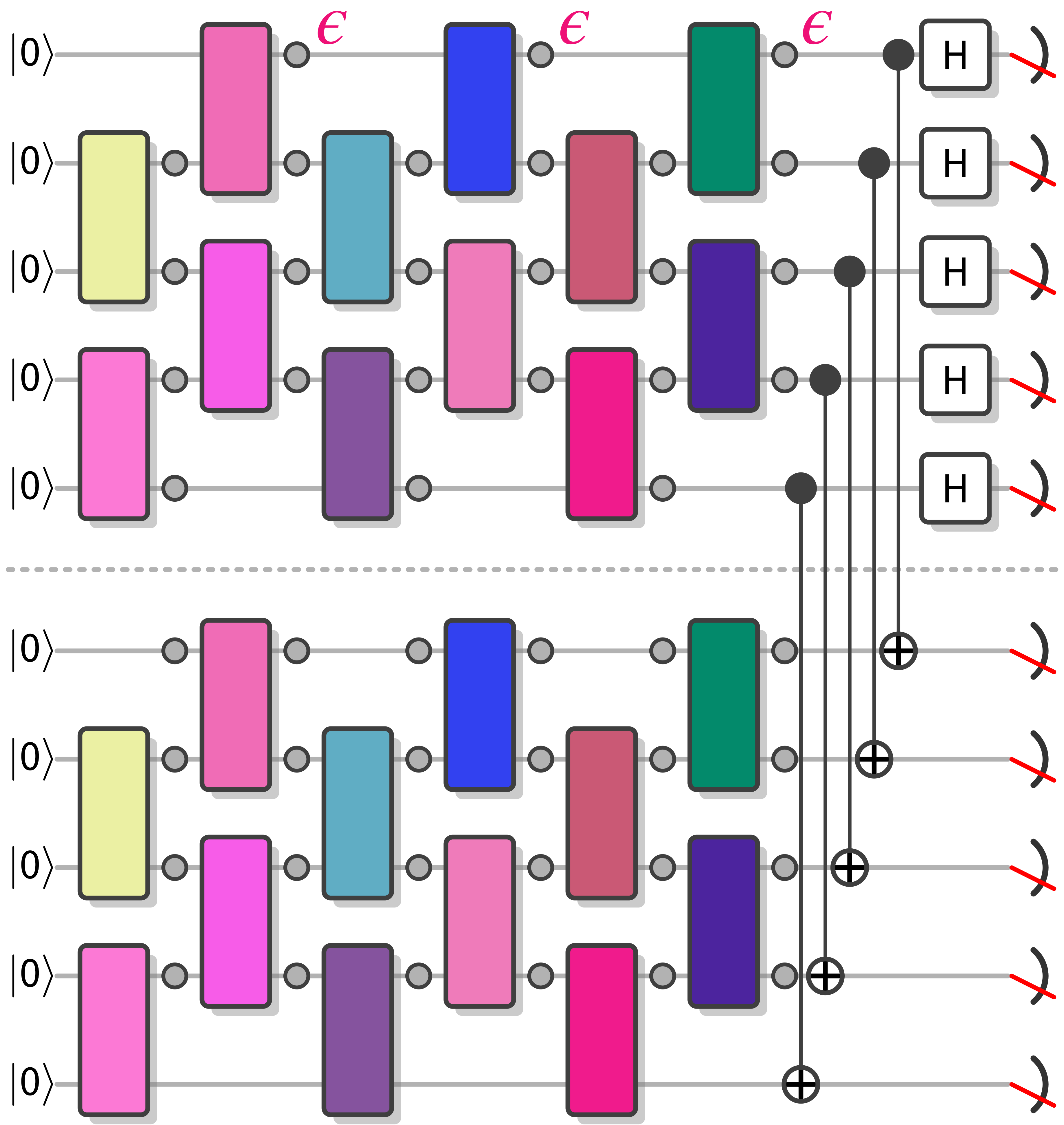
$\epsilon < 1/n$ → $$\rho_C(\eta) = (1-\eta)|C\rangle\langle C| + \eta\frac{\mathbb{1}}{2^n}$$

→ Ideal measurement

Scalability

---> the constant-noise regime

On robustness to noisy simulation algorithms



$\epsilon \in \Omega(1)$

→ Efficient classical algorithm can achieve $1/\mathrm{poly}(n)$ TVD to noisy quantum distribution with **constant** & **depolarizing** & **local** noise.

Bremner, Montanaro, Shepherd, 2017
Gao & Duan, 2018; Aharonov *et al.*, 2022

→ Truncated Pauli path distribution seems to be exponentially far from the noisy distribution

→ All Paulis contribute to the distribution!

Is the quantum advantage in Bell sampling more noise-robust?

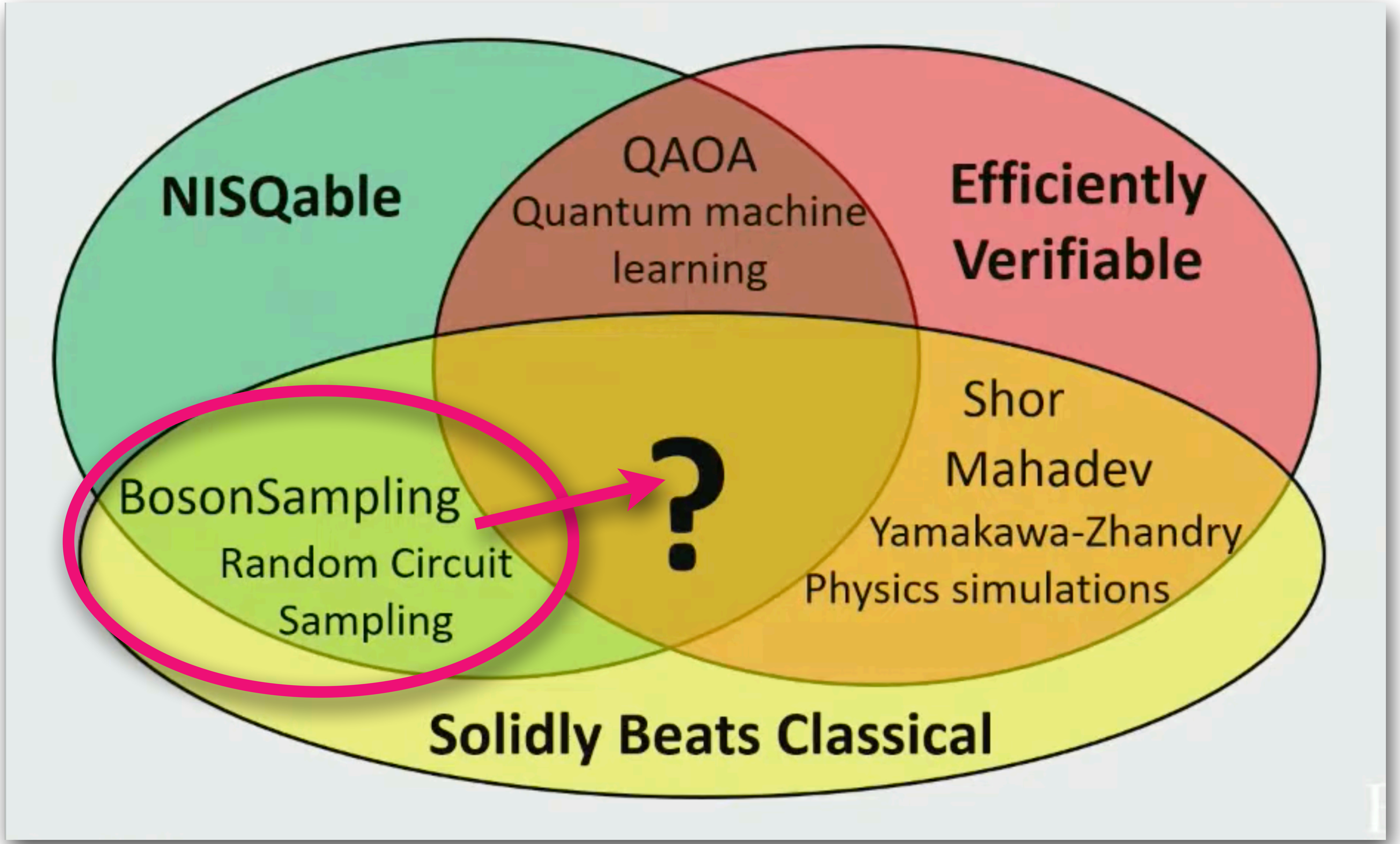
Summary & outlook

Quantum advantage demonstrations

Is there a

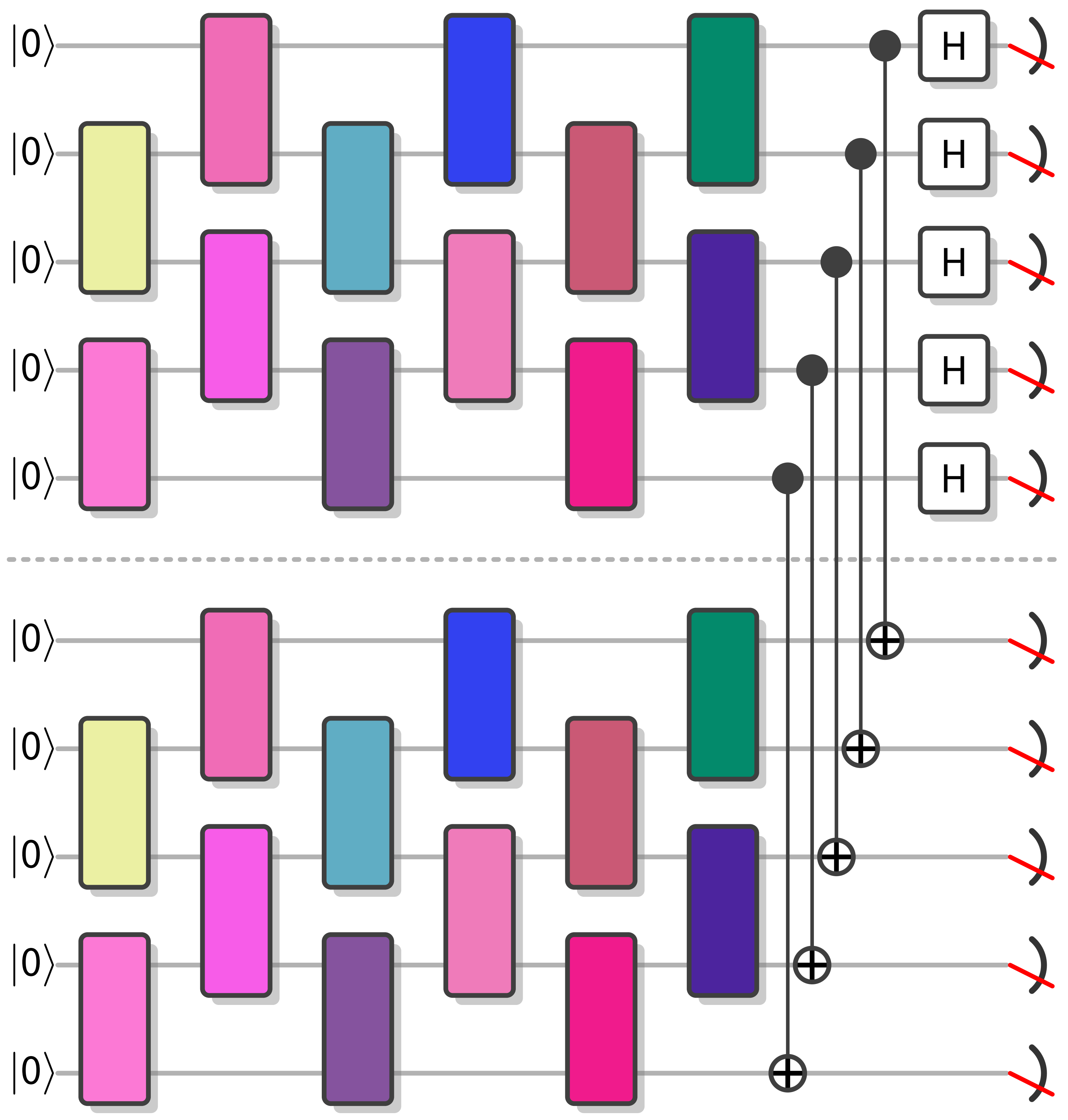
- Scalable
- Verifiable
- NISQ

Quantum advantage?



Aaronson, Simons Workshop 2022

Bell sampling highlights



→ Hardness of sampling

→ Structured distribution with efficient & inefficient property tests

→ Efficient fidelity estimation in the same regime in which XEB works

→ Quantum advantage test that is more difficult to spoof than just XEB (?)

→ More noise robust than standard-basis sampling (?)

Outlook & questions

Noise-robustness of Bell sampling?

Hardness of achieving high test scores?

Is Bell sampling a universal model of quantum computation?

How make the best possible use of the samples in the tests?

More error mitigation with more copies?