Joint Center for Quantum Information and Computer Science

University of Maryland 1856

Is there an advantage of quantum learning?

Dominik Hangleiter

Quantum Workshop NCSU, January 22, 2020

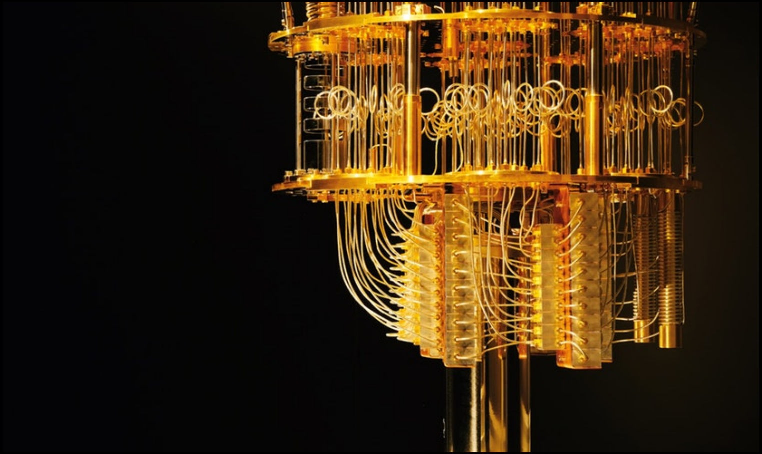
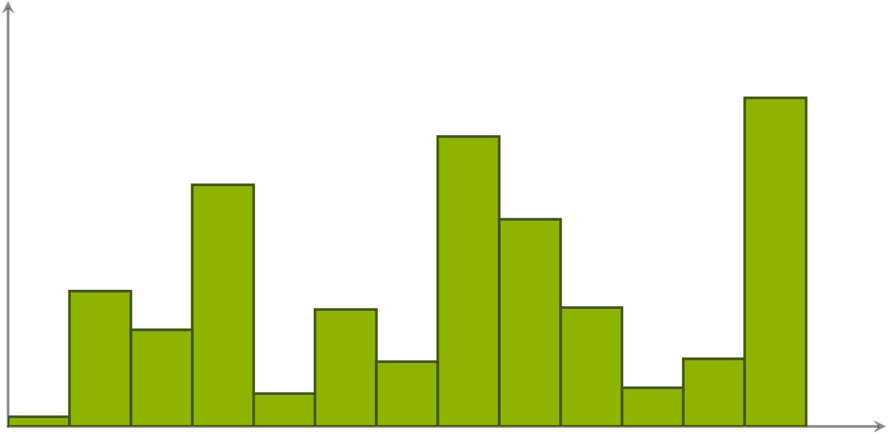


Image Ref.: Graham Carlow/IBM

vs.



Machine learning

Image Ref.: Wikipedia – Katze (CC0)

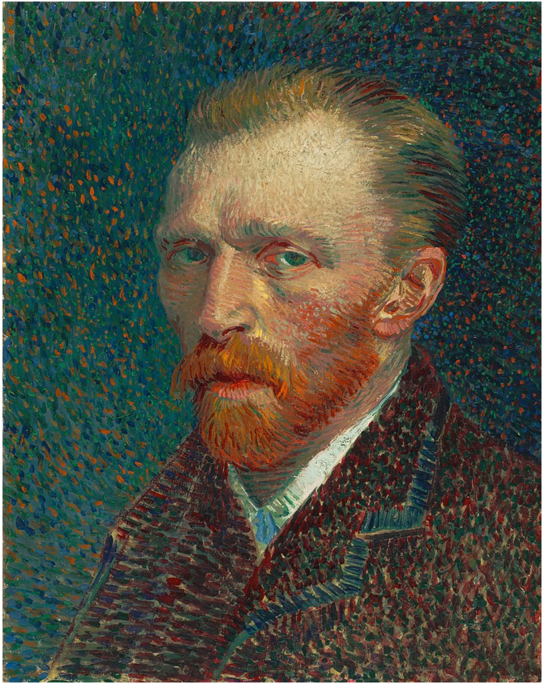
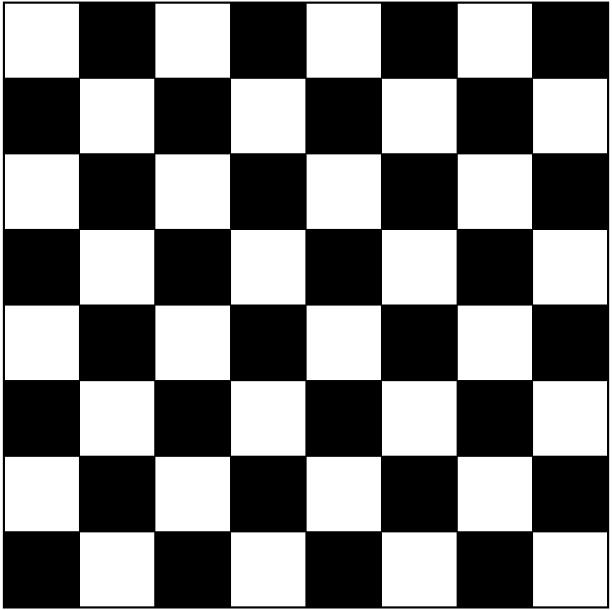


Image Ref.: Wikipedia – Vincent Van Gogh (CC0)



Machine learning and distribution learning

Supervised learning

Unsupervised learning

Reinforcement learning

oracle

learner

Image Refs.: Wikipedia – Hund, Katze, Van Gogh, Kaffeesatz (CC0), QC: Graham Carlow/IBM

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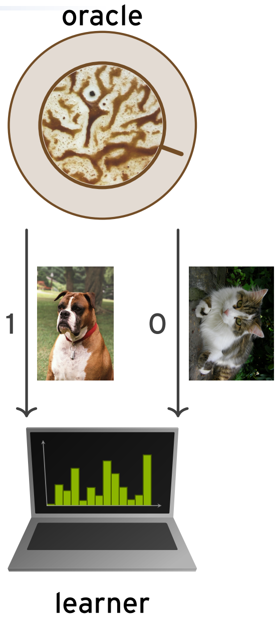

Machine learning and distribution learning

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Distribution on {images} $\times \{0, 1\}$.

→

→ ?

learner

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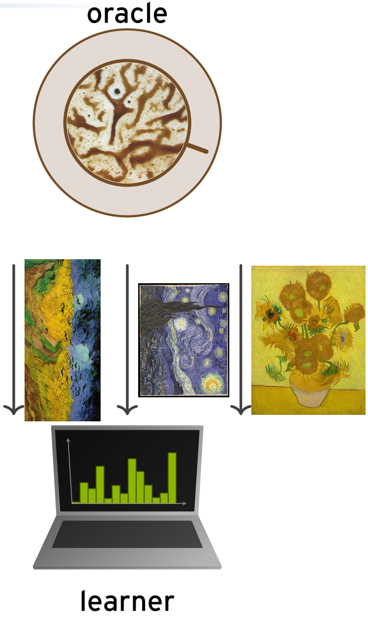

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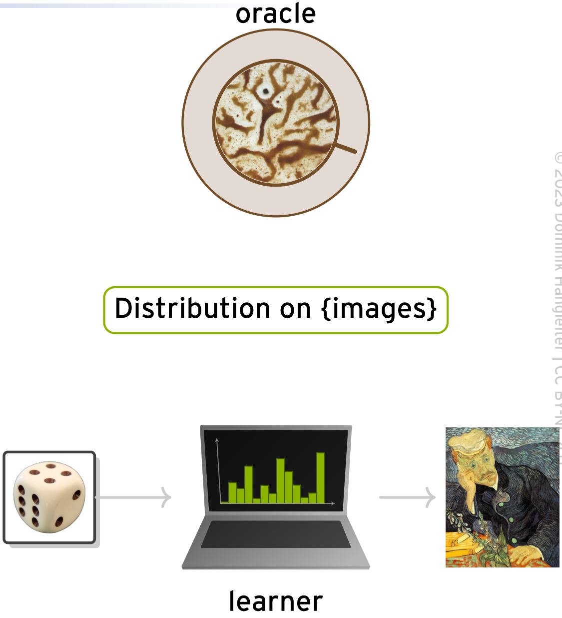

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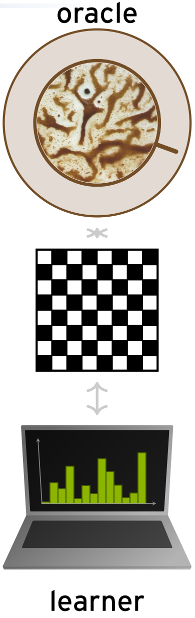

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Distribution on moves,
conditioned on environ. configs.

learner

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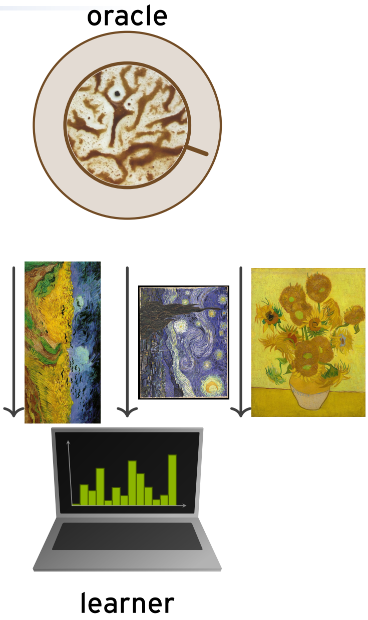

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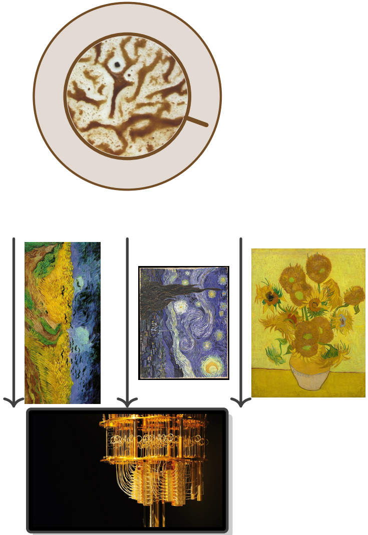
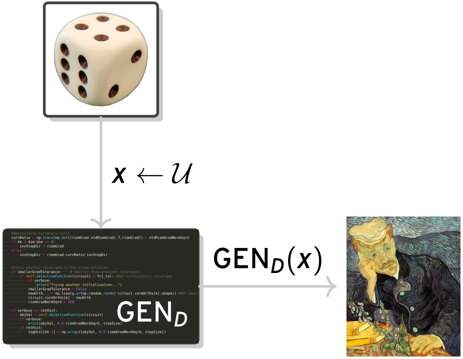


quantum learner

Unsupervised learning: generator vs. evaluator learning



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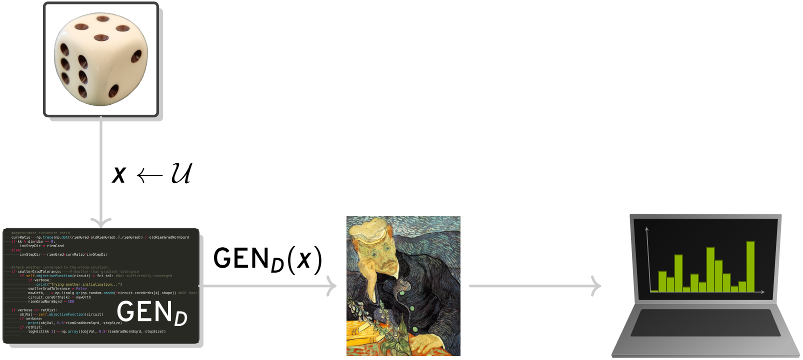

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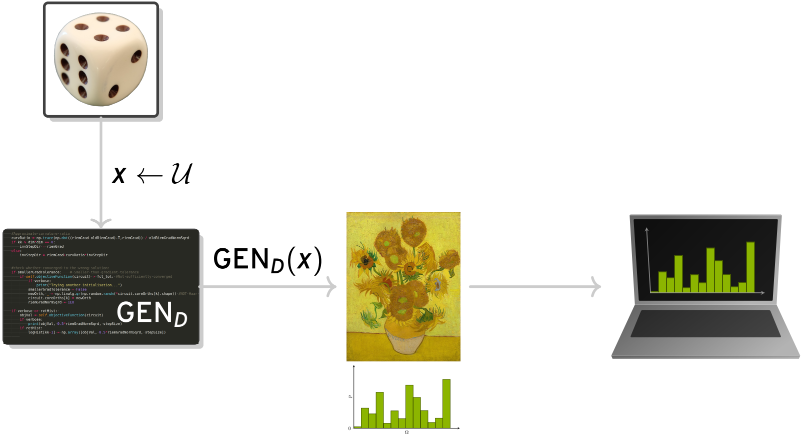

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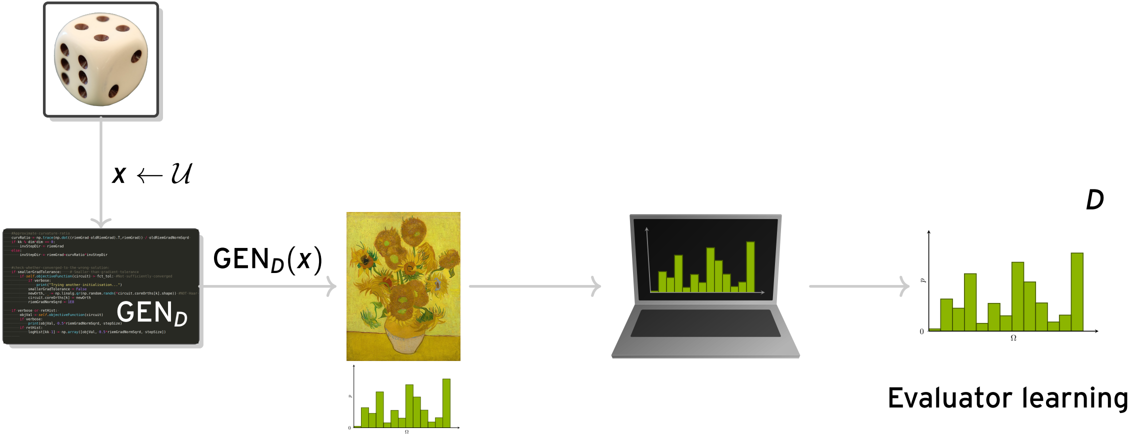

Task: Learn an evaluator EVAL$_D$ of a distribution D.

Unsupervised learning: generator vs. evaluator learning

Task: Learn a generator GEN_D of a distribution D.

The details matter – the case of function learning

Learning Boolean functions $f : \{0,1\}^n \to \{0,1\}$.

Arunachalam and de Wolf: *A survey of quantum learning theory.* SIGACT news (2017).
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exact learning

PAC learning

agnostic learning

membership queries

random example queries

quantum example queries

query complexity

sample complexity

computational complexity

no speedup

at most poly speedup

super-poly speedup

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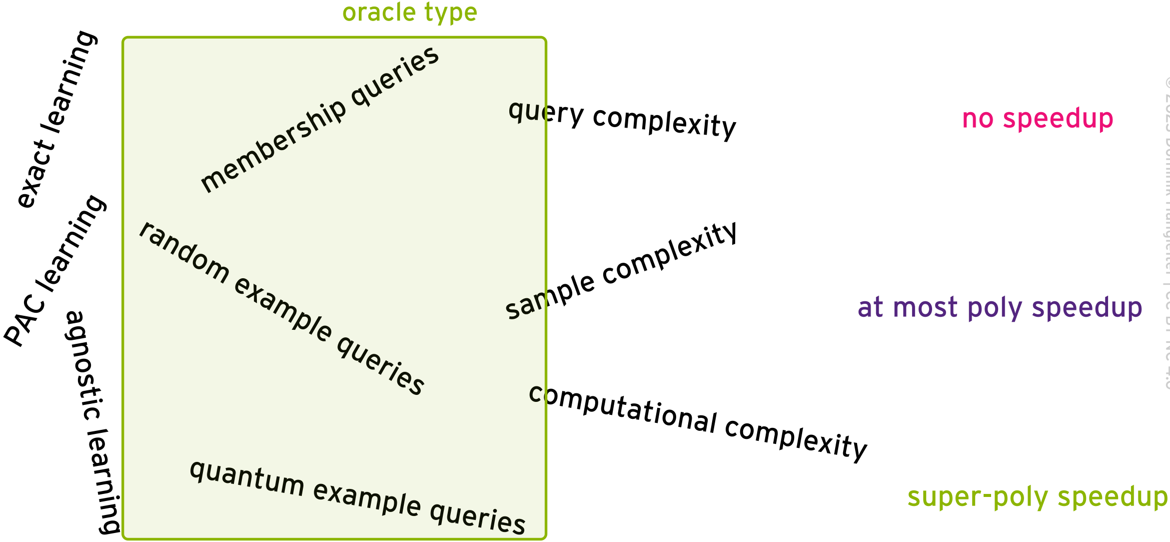

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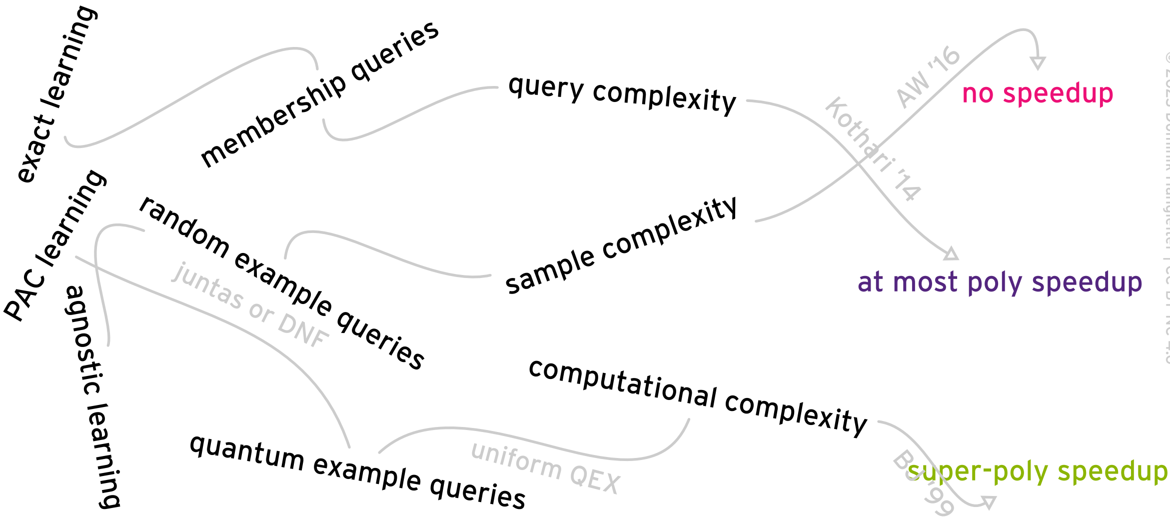

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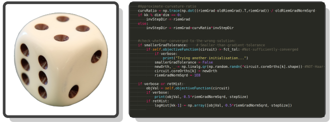
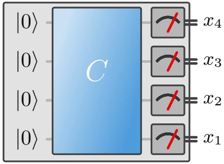
Learning settings: classical vs. quantum

Generators



GEN_D



QGEN_D

Oracle access

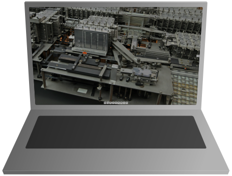
SAMPLE(D)

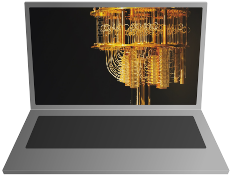
$x \leftarrow D$

QSAMPLE(D)

$\sum_x \sqrt{D(x)}|x\rangle$

Learning algorithm





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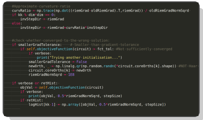

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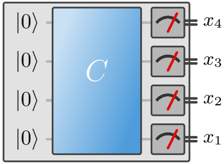

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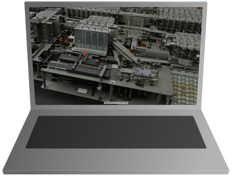

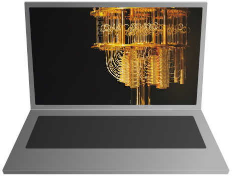

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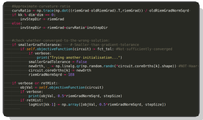

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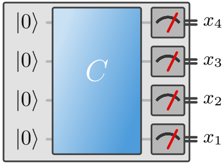

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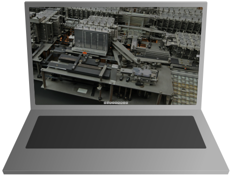

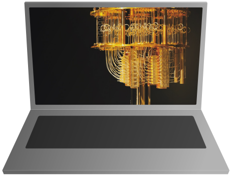

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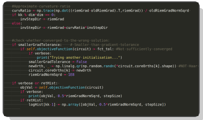

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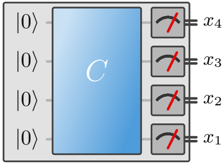

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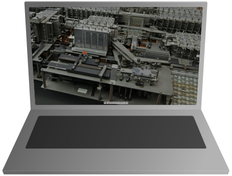

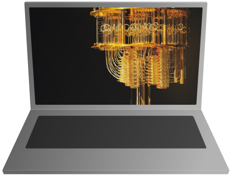

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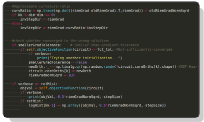

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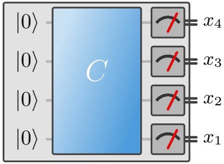

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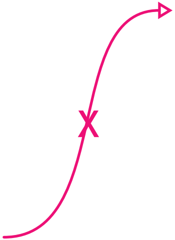

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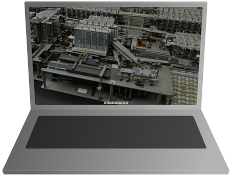

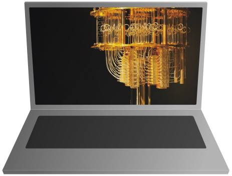

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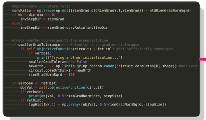

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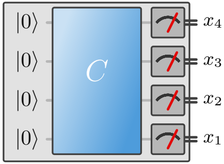

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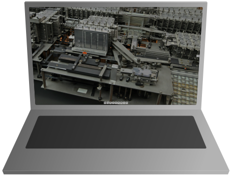

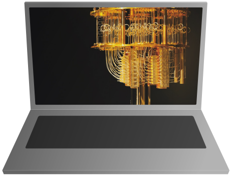

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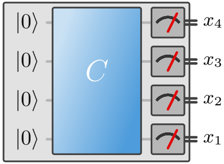

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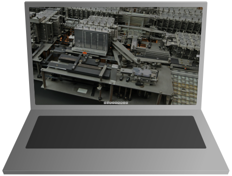

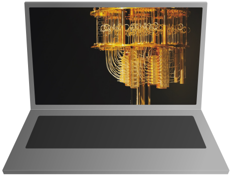

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Is there a quantum advantage in generator learning?

Generator learning

Task: Learn a generator GEN_D of a distribution D.

A quantum vs. classical separation for distribution learning

Question: Quantum generator-learning advantage?

Is there a class of *efficiently classically generated* discrete distributions which is

- not efficiently classsical PAC generator-learnable, but
- efficiently quantum PAC generator-learnable

w.r.t. the *SAMPLE oracle* and the *KL divergence*?

Coyle *et al.*, NPJ Quant. Inf. (2020).

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Theorem: Y E S* !

*under the decisional Diffie-Hellman assumption for the group family of quadratic residues

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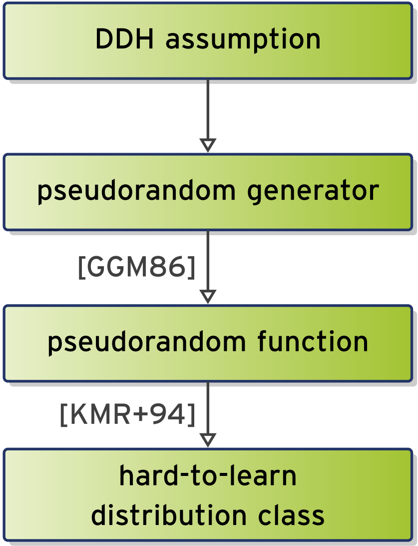
Proof idea: classical hardness and quantum easiness



[KMR+94] Kearns *et al.*, STOC '94 [GGM86] Goldwasser, Goldreich, Micali J. ACM '86 [MZ03] Mosca, Zalka '03

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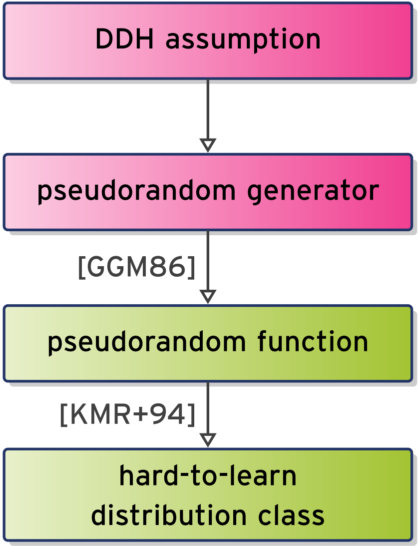

- g generator of $(G, \cdot)$ of order q.
- Think: $\mathbb{Z}_q^*$ and $\text{modexp}_{q,g}(x) = g^x \bmod q$

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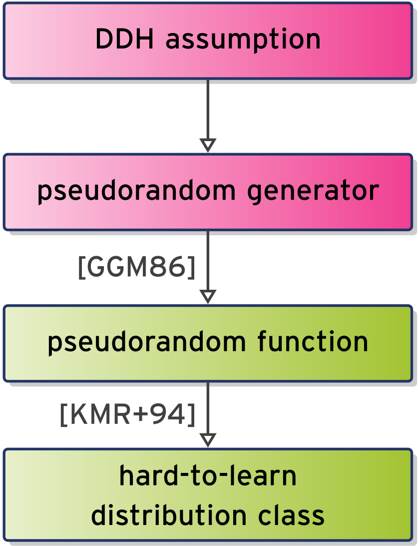

Proof idea: classical hardness and quantum easiness

DDH assumption

↓

pseudorandom generator

↓ [GGM86]

pseudorandom function

↓ [KMR+94]

hard-to-learn distribution class

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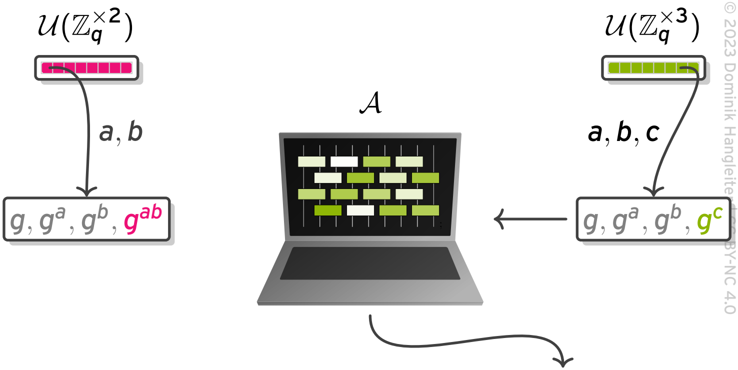

$$\left| \Pr_{a,b \leftarrow \mathcal{U}(\mathbb{Z}_q)}[\mathcal{A}(g, g^a, g^b, g^{ab}) = 1] - \Pr_{a,b,c \leftarrow \mathcal{U}(\mathbb{Z}_q)}[\mathcal{A}(g, g^a, g^b, g^c) = 1] \right|$$

problem size — problem size

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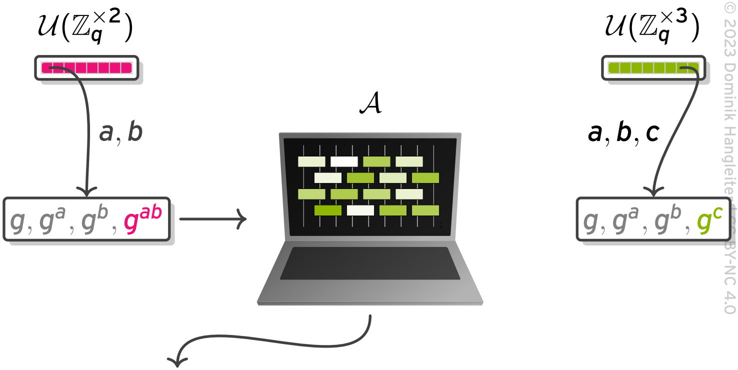


$$\left| \Pr_{a,b \leftarrow \mathcal{U}(\mathbb{Z}_q)}[\mathcal{A}(g, g^a, g^b, g^{ab}) = 1] - \Pr_{a,b,c \leftarrow \mathcal{U}(\mathbb{Z}_q)}[\mathcal{A}(g, g^a, g^b, g^c) = 1] \right|$$

problem size　　problem size

Proof idea: classical hardness and quantum easiness

DDH assumption

↓ quantum adversary

pseudorandom generator

[GGM86] ↓ quantum adversary

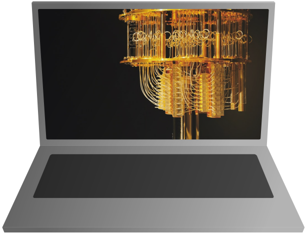
pseudorandom function

[KMR+94] ↓ quantum adversary

hard-to-learn
distribution class

dlog

[MZ03]



[KMR+94] Kearns *et al.*, STOC '94

[GGM86] Goldwasser, Goldreich, Micali J. ACM '86 [MZ03] Mosca, Zalka '03

Proof idea: classical hardness and quantum easiness

DDH assumption

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pseudorandom ge

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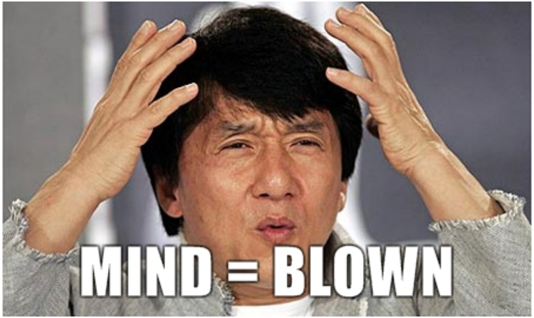
pseudorandom fu

[KMR+94]

qu

hard-to-lear
distribution class

[MZ03]



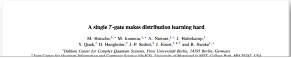
The distribution class can be exactly generator-learned from a single sample!

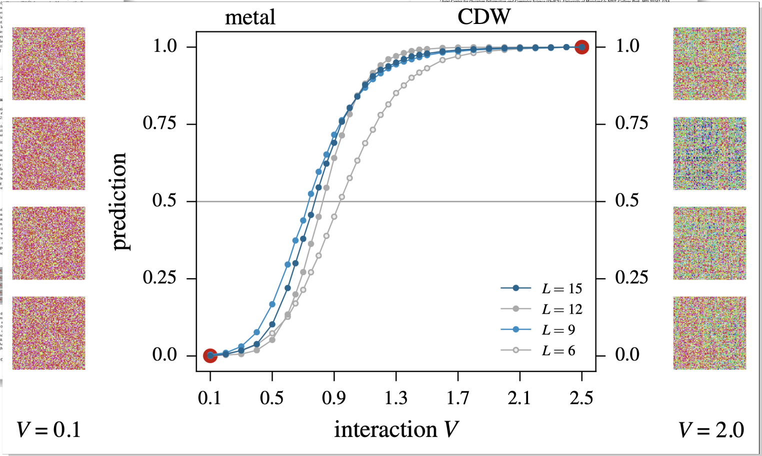
[KMR+94] Kearns *et al.*, STOC '94
[GGM86] Goldwasser, Goldreich, Micali J. ACM '86 [MZ03] Mosca, Zalka '03

A more generic setting

Relevant distributions

Relevant distributions

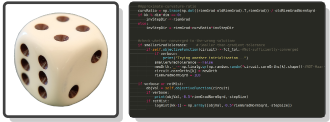




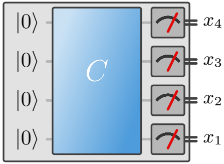
[Broecker *et al.*, 2017]

Learning settings: classical vs. quantum

Generators



GEN_D



QGEN_D

Oracle access

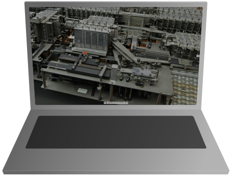
SAMPLE(D)

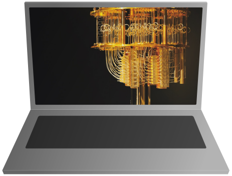
$x \leftarrow D$

QSAMPLE(D)

$\sum_x \sqrt{D(x)}|x\rangle$

Learning algorithm



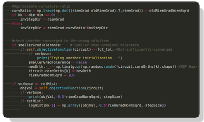


Excellent discussion: Bshouty and Jackson, SIAM J. Comput. (1998)

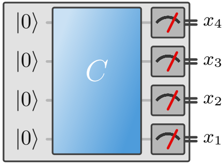
Figure Refs. Dice, coffee grounds: Wikipedia (CC0), QC: Graham Carlow/IBM

Learning settings: classical vs. quantum

Generators



GEN$_D$



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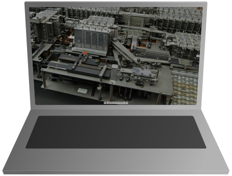

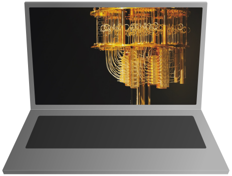

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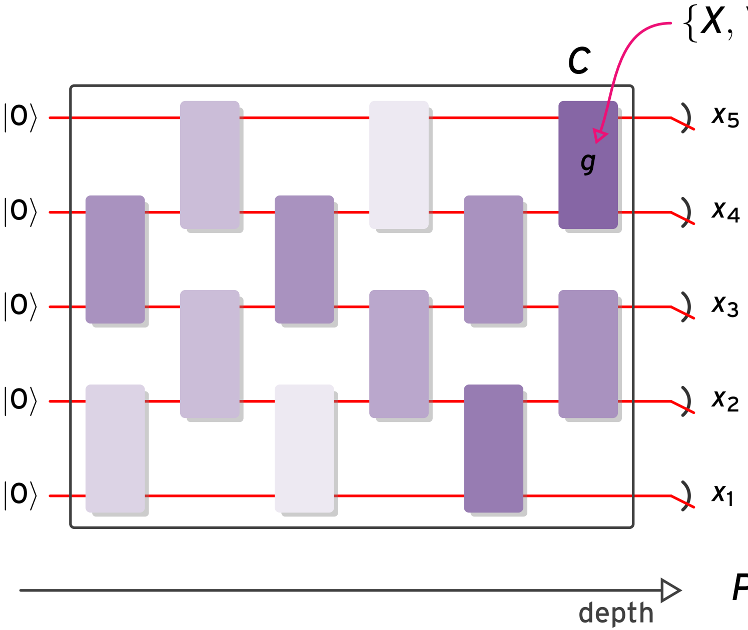
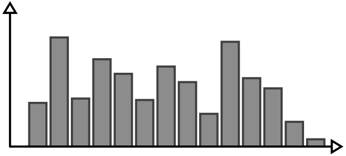
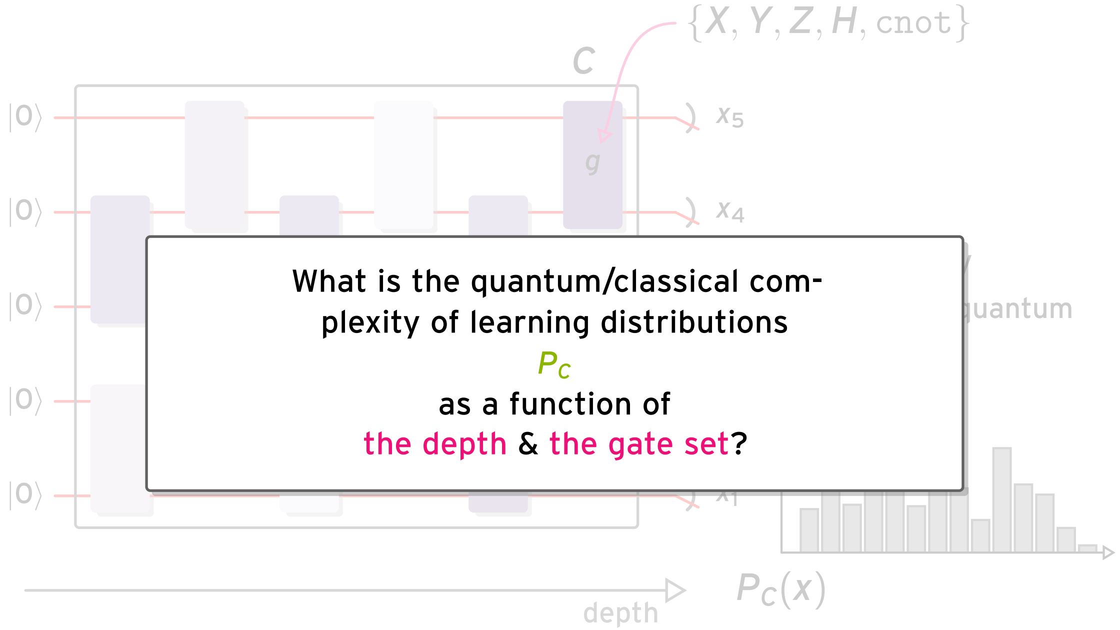
Output distributions of quantum circuits



- expressivity
- efficiently quantum generated



Output distributions of quantum circuits



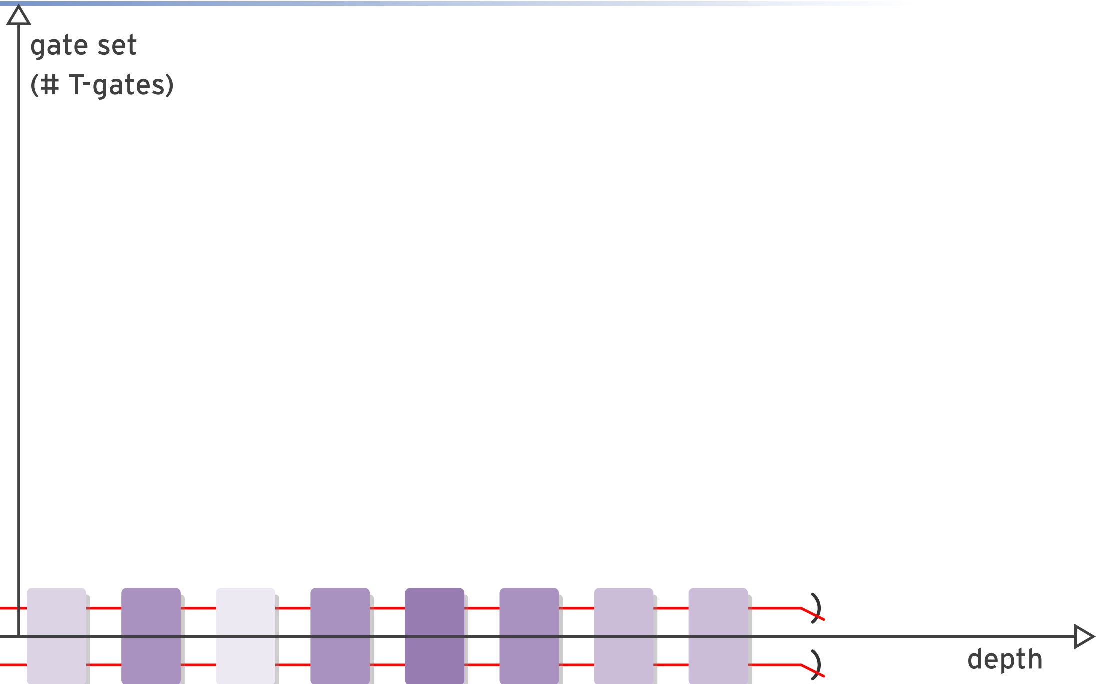
What is the quantum/classical complexity of learning distributions

P_C

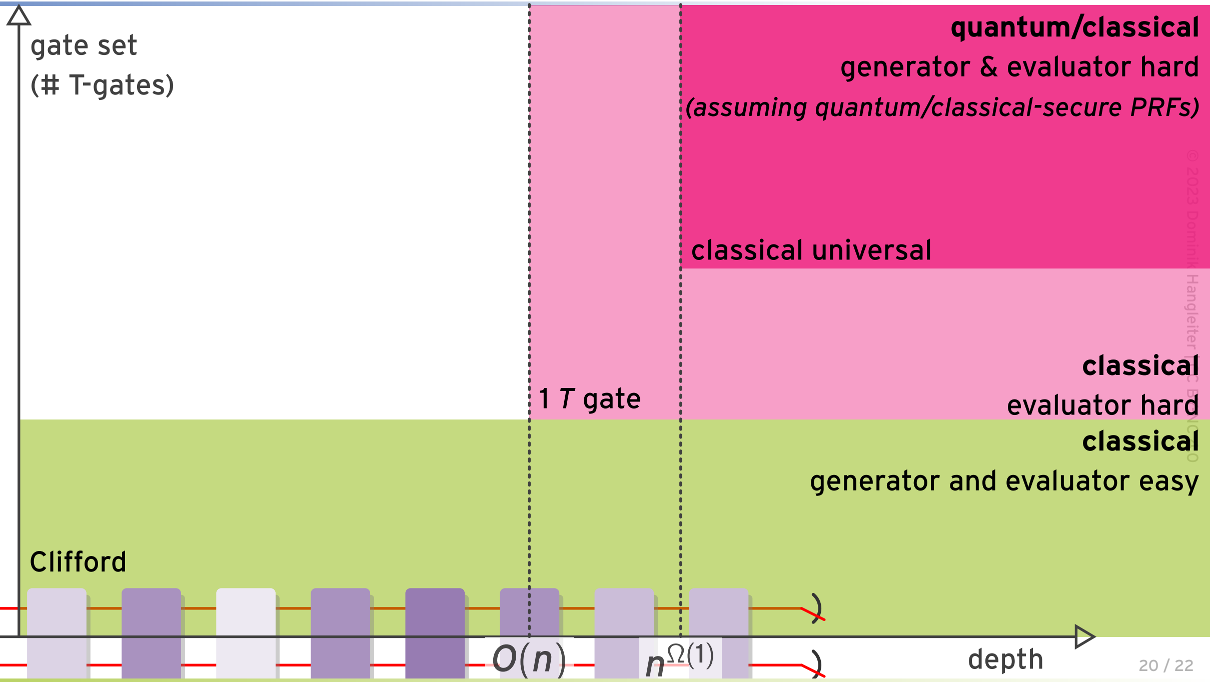
as a function of

the depth & the gate set?

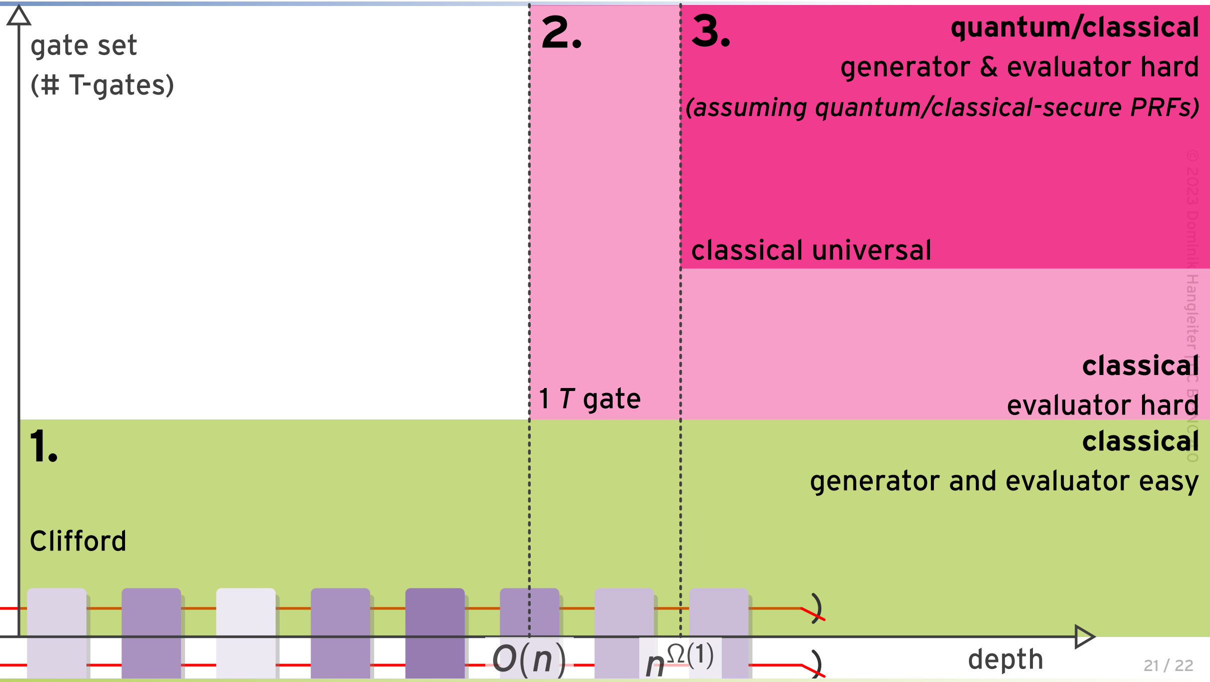
Our results



Our results


gate set
(# T-gates)
quantum/classical
generator & evaluator hard
(assuming quantum/classical-secure PRFs)
classical universal
1 T gate
classical
evaluator hard
classical
generator and evaluator easy
Clifford
$O(n)$
$n^{\Omega(1)}$
depth

Our results: Proof idea



Our results: Proof idea

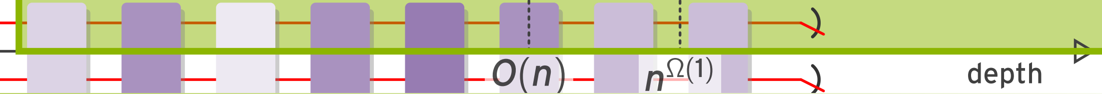
$$\forall \text{ Cliffords } C : P_C(x) = \begin{cases} 2^{-n} & \text{if } x = Rb + t,\ b \in \mathbb{F}_2^m \\ 0 & \text{else} \end{cases}$$

- Sample $O(n)$ strings $x_0, \ldots, x_k \leftarrow P_C$
- Find a basis of span(R) using Gaussian elmination with $y_i = x_0 + x_i$.

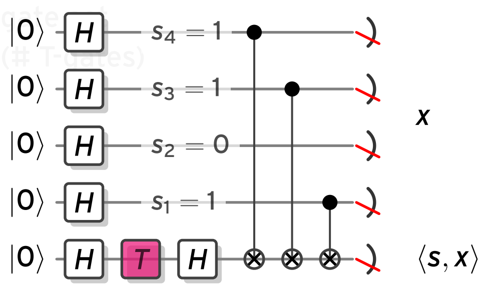
1. **classical**

generator and evaluator easy

Clifford



Our results: Proof idea



2.

3. **quantum/classical**
generator & evaluator hard
(assuming quantum/classical-secure PRFs)

classical universal

classical
evaluator hard

$$P_C(y) = \begin{cases} (1 - \sin^2(\frac{\pi}{8}))/2^n & \text{if } y = (x, \langle s, x \rangle) \\ \sin^2(\frac{\pi}{8})/2^n & \text{if } y = (x, \overline{\langle s, x \rangle}) \end{cases}$$

→ An efficient evaluator would be able to efficiently solve the **learning parity with noise** problem.

Our results: Proof idea

3. **quantum/classical**
generator & evaluator hard
(assuming quantum/classical-secure PRFs)

classical universal

Theorem 17 [KMRRSS94] Polynomial-size classical circuits are hard to learn with respect to a generator or an evaluator if a pseudorandom function (PRF) exists.

⟶ Quantum circuits can implement classical circuits.

[KMRRSS94] Kearns, Mansour, Ron *et al.*, On the learnability of discrete distributions (1994).

Discussion

SUMMARY

- Assessing the power of quantum learning is intricate!

OUTLOOK

- Learnability of low-depth circuits.
- Learnability of quantum samples.
- Is there an advantage for a relevant problem, e.g., learning *mixtures of Gaussians*?

Discussion

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arXiv:2007.14451
arXiv:2207.03140

SUMMARY

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THANK YOU!