Joint Center for Quantum Information and Computer Science

University of Maryland 1856

HIDING SECRETS IN IQP CIRCUITS

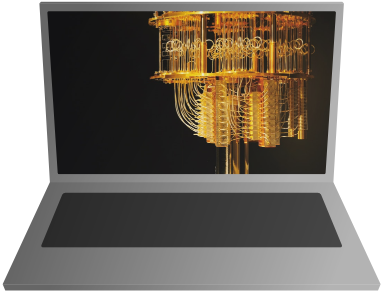
A drama in three acts

Dominik Hangleiter

with David Gross

Arlington, June 11, 2024

Verifying quantum advantage



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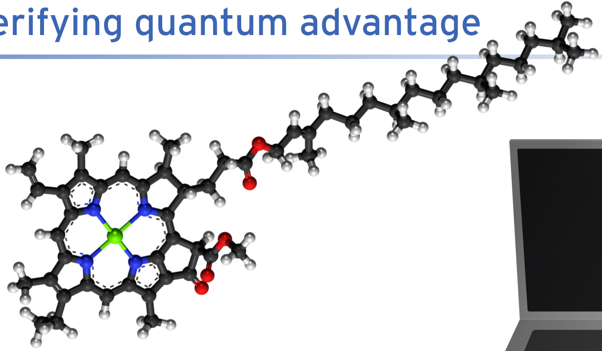

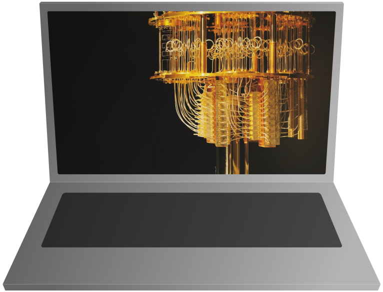

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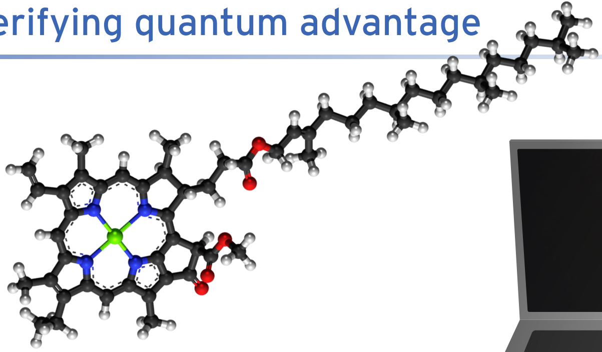

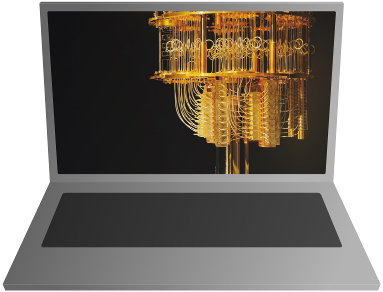

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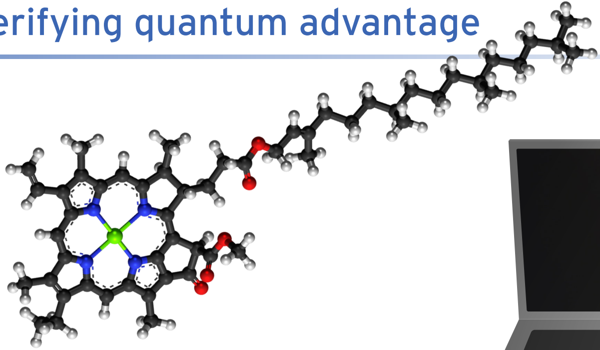

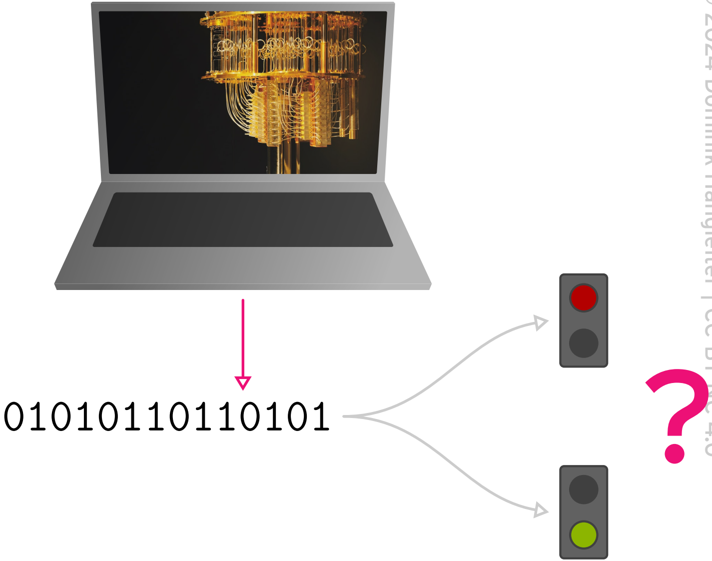

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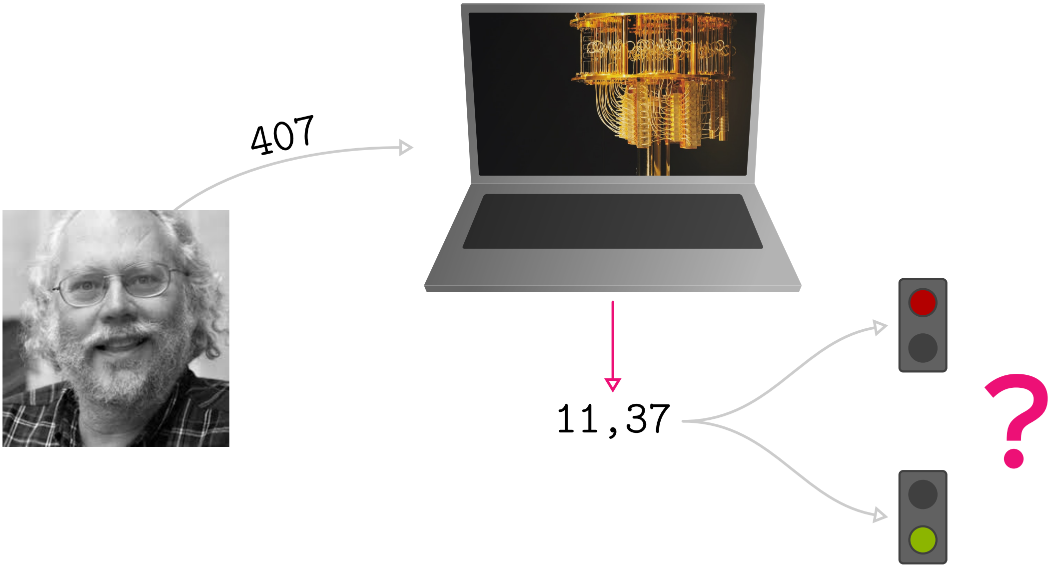
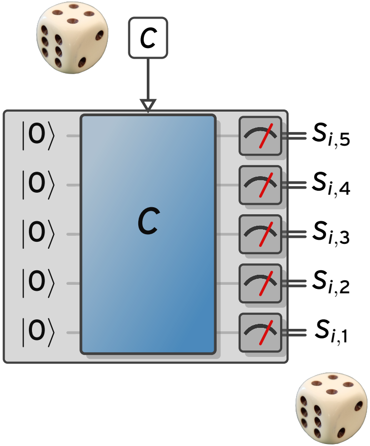


Quantum random sampling

 $\boxed{C} \in \{C_0, \ldots, C_N\}$

Quantum random sampling



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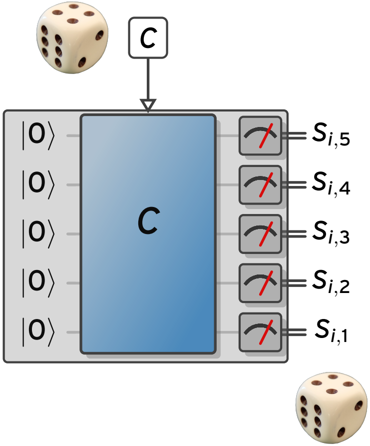
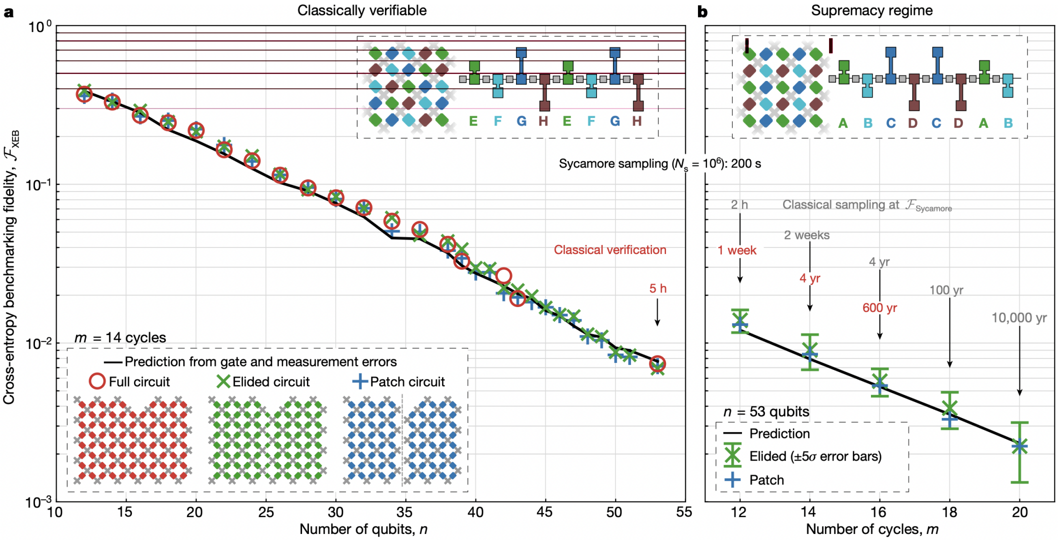


Classical simulations are *provably* inefficient.

Review: DH, Eisert, RMP (2023)

Verifying quantum sampling



Arute *et al*, Nature (2019)

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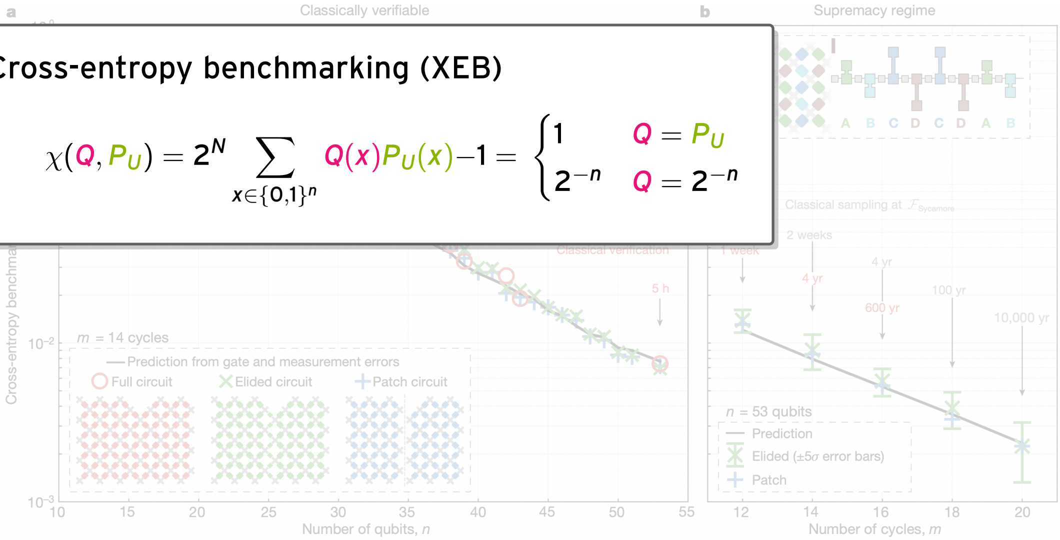

Cross-entropy benchmarking (XEB)

$$\chi(Q, P_U) = 2^N \sum_{x \in \{0,1\}^n} Q(x) P_U(x) - 1 = \begin{cases} 1 & Q = P_U \\ 2^{-n} & Q = 2^{-n} \end{cases}$$

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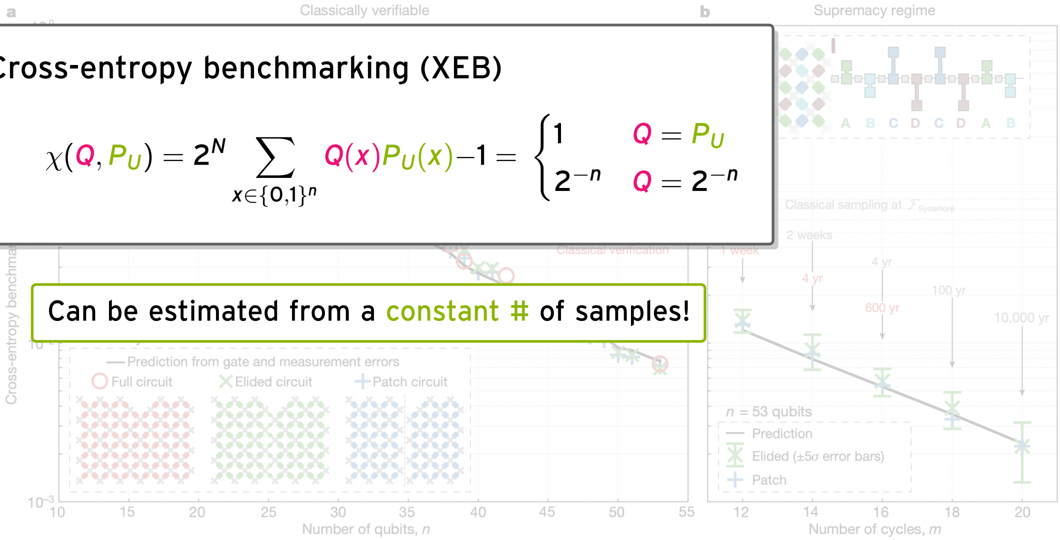

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Can be estimated from a constant # of samples!

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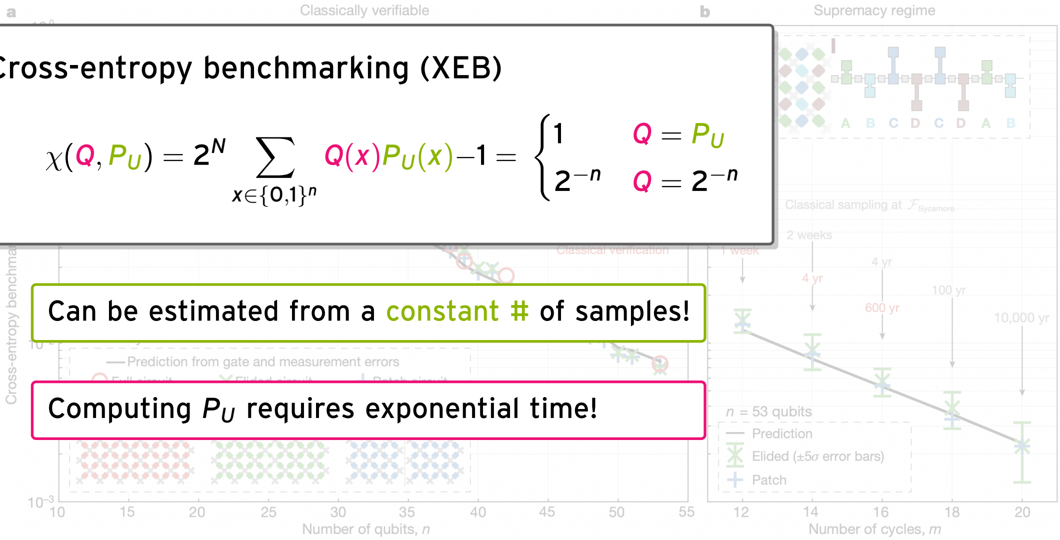

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Computing P_U requires exponential time!

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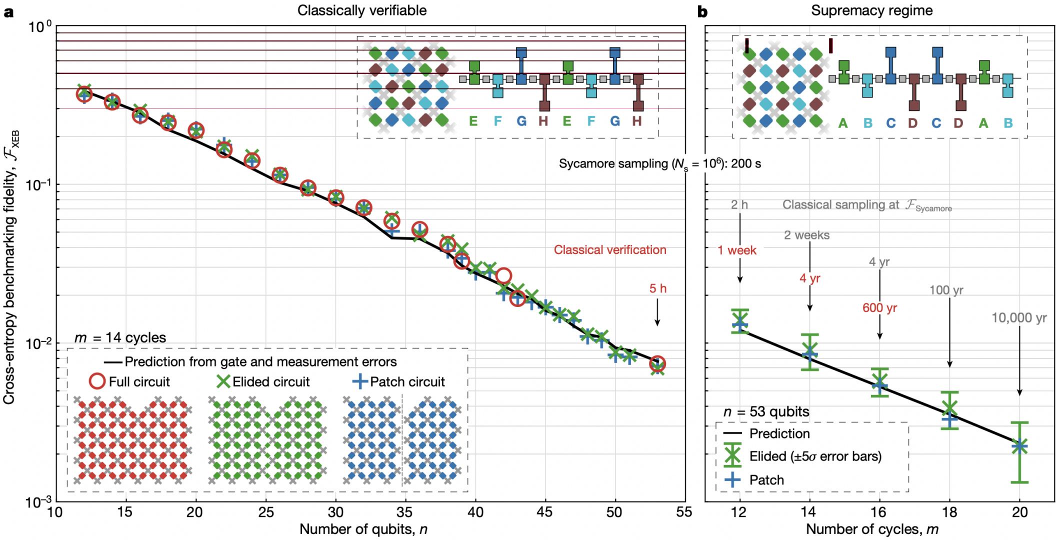

Arute *et al*, Nature (2019)

Can we efficiently verify quantum sampling?

ACT I

Dan and Mick have an idea

X-programs

X program [SB09]

- Angle θ
- $\mathbf{P} \in \{0,1\}^{m \times n}$
- $H_{\mathbf{P}} = \sum_i \left(\prod_j X_j^{\mathbf{P}_{i,j}} \right)$

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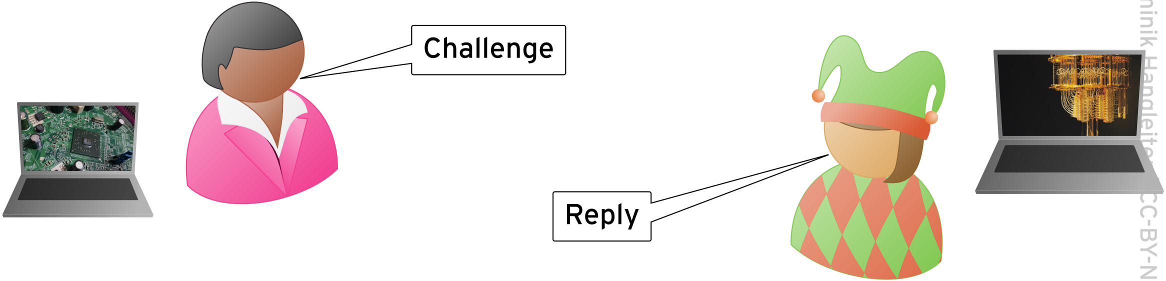
Example

$$\mathbf{P} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \qquad H_{\mathbf{P}} = X_2X_3 + X_1X_2X_3 + X_1X_2X_3X_4 + X_2 + X_1X_3X_4$$

[SB09] Shepherd & Bremner (2009) [She10] Shepherd, arXiv:1005.1744 [BCJ23] Bremner, Cheng, Ji, arXiv:2308.07152.

Hiding a secret in the output of an X-program



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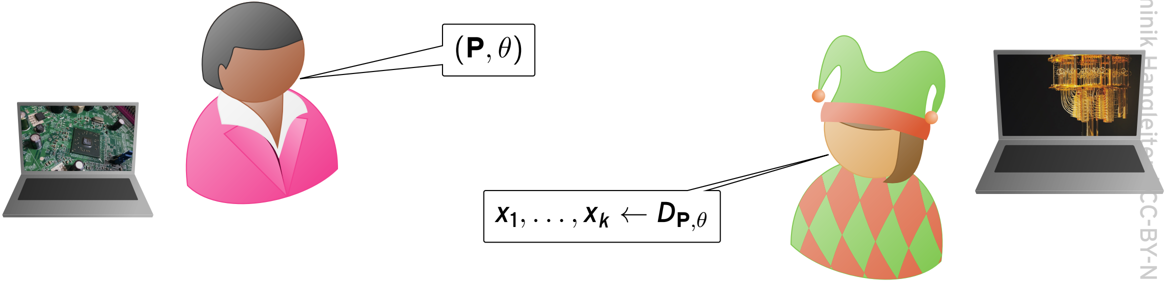
→ Design an X program $(\mathbf{P}, \theta)$ with a secret s.

Prepare $e^{i\theta H_{\mathbf{P}}}|0\rangle$ and measure. ←

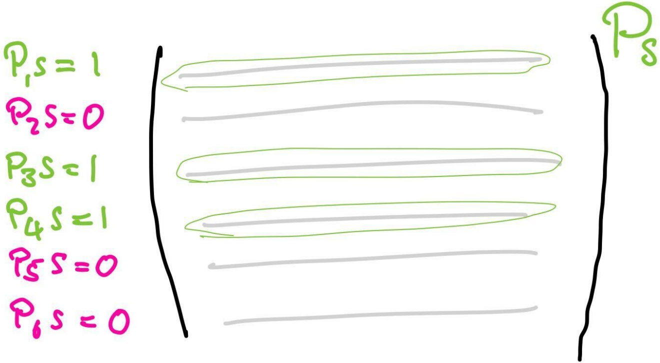


→ Compute $\frac{1}{k}\sum_k x_k \cdot s \approx \Pr[x \cdot s = 0]$.

Dan and Mick's tricks

The double angle trick [SB09,She10]
Fourier coefficients are given by the zero-amplitude of a different *X*-program with double angle

$$\beta_s = \langle Z_s \rangle = \langle 0 | e^{i 2\theta H_{\mathbf{P}_s}} | 0 \rangle, \qquad \text{where } (\mathbf{P}_s)_i = \mathbf{P}_i \text{ iff } \mathbf{P}_i \cdot s = 1.$$



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- For $\theta = \pi/4$, an X-program is a Clifford circuit.
- Can compute Fourier coefficients for hard circuits with $\theta = \pi/8$.
- Sampling from random X programs with $\theta = \pi/8$ is classically hard.

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The coding theory trick [SB09,She10]

$$\langle 0 | e^{i\pi/4 H_{\mathbf{P}}} | 0 \rangle = \begin{cases} 2^{-\operatorname{rank}(\mathbf{P}^T\mathbf{P})/2} & \operatorname{col}(\mathbf{P}) \cap \operatorname{col}(\mathbf{P})^\perp \text{ is doubly even} \\ 0 & \text{else} \end{cases}$$

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For random $\mathbf{P}$, $\mathrm{rank}(\mathbf{P}^T\mathbf{P}) \sim n$

→ For most $\mathbf{s}$, $\beta_{\mathbf{s}} \lesssim 2^{-n}$

…Clifford circuit.

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Goal
Design $\mathbf{P}$ such that $\mathbf{P}_s^T\mathbf{P}_s$ has large kernel for a secret $\mathbf{s}$.

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→ Understand $\operatorname{rank}(\mathbf{P}_s^T \mathbf{P}_s)$

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Interlude: Geometry of the problem

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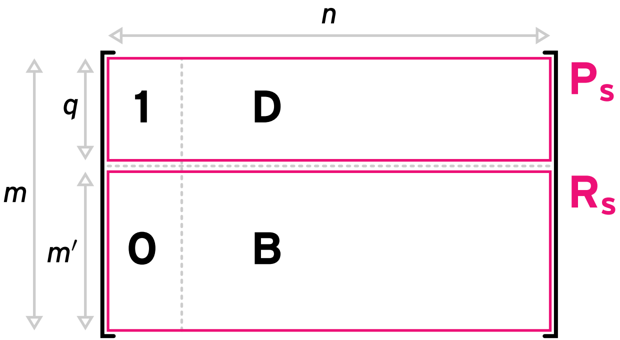
→ $\mathbf{P}_\mathbf{s}^T \mathbf{P}_\mathbf{s}$ is the Gram matrix describing the geometry of $\operatorname{col}(\mathbf{P}_\mathbf{s})$

→ $\mathbf{d} \in \ker \mathbf{P}_\mathbf{s}^T \mathbf{P}_\mathbf{s} \quad \Leftrightarrow \quad \mathbf{d} \in \operatorname{rad} \operatorname{col}(\mathbf{P}_\mathbf{s})$,

Radical of vector space V: $\operatorname{rad}(V) = V \cap V^\perp$

Hiding a secret in the output of an X-program



Hiding a secret in the output of an X-program

$$\mathbf{P} = \left[\begin{array}{c:c} \mathbf{1} & \mathbf{D} \\ \hdashline \mathbf{0} & \mathbf{B} \end{array}\right] \cdot \mathbf{X}$$

$$\boldsymbol{s} = \left[\begin{array}{c:c} 1, & 0, 0, \ldots \end{array}\right]$$

→ [BS09] choose **D** as a quadratic residue code (radical is doubly even).

Hiding a secret in the output of an X-program

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→ The output distribution of $(\mathbf{P}, \pi/8)$ has $\beta_{\mathbf{s}} = 1/\sqrt{2}$

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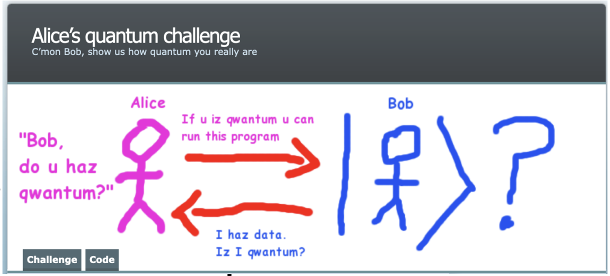

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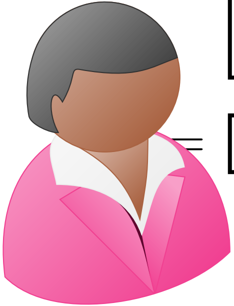

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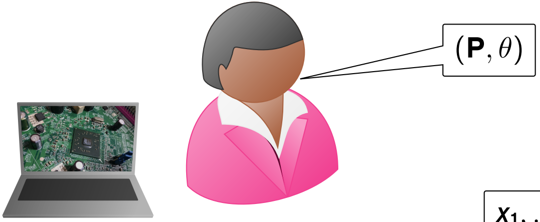

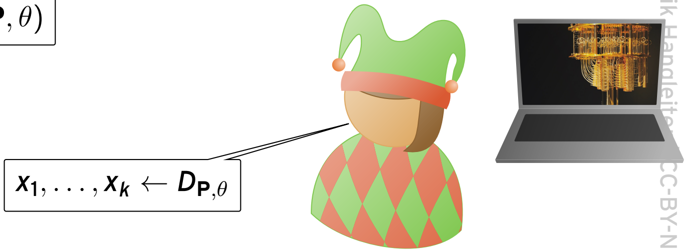

→ Compute $\frac{1}{k}\sum_k x_k \cdot s \approx \Pr[x \cdot s = 0]$.

ACT II

Greg is a killjoy but IQP comes back

The Kahanamoku-Meyer attack

Greg's trick [Kah19]

$$\text{For } d \in \mathbb{F}_2^n : \mathbf{P}_s d \in \operatorname{rad}(\operatorname{col}(\mathbf{P}_s)) \quad \Rightarrow \quad s \in \ker(\mathbf{P}_d^T \mathbf{P}_d).$$

[Kah19] Kahanamoku-Meyer arXiv:1912.05547 [GH23] Gross & DH, arXiv:2312.10156

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Attack

1 Draw d randomly.

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- With probability $2^{-\mathrm{rank}(\mathbf{P}_s^T \mathbf{P_s})}$ s lies in $\ker \mathbf{P_d}^T \mathbf{P_d}$.
- For the [SB09] QRC construction $\ker \mathbf{P_d}^T \mathbf{P_d}$ is typically small ($2^{n-m/2}$ elements).

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The SECRET is Bi1bHzjYxr0HYH401EJFBoXZbps4a54kH8f1rRgo/g==

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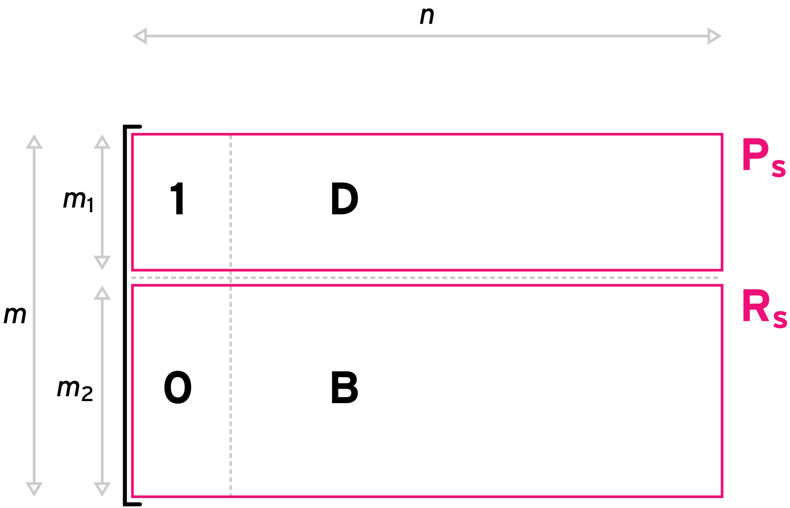

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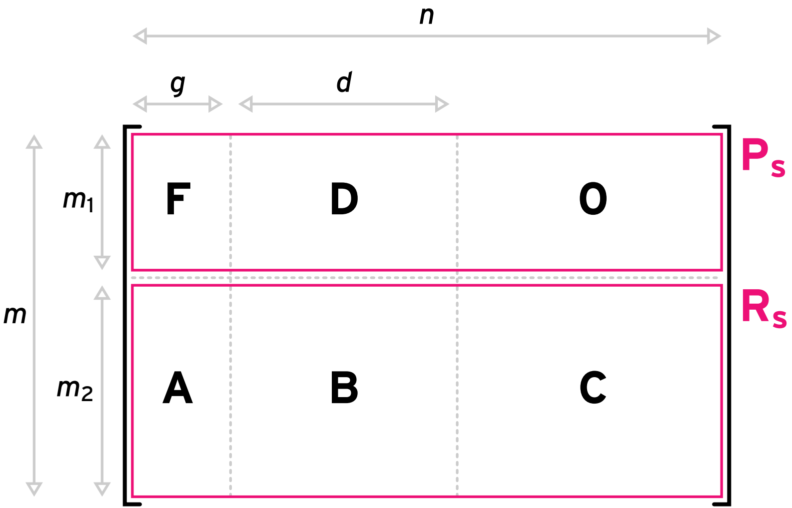

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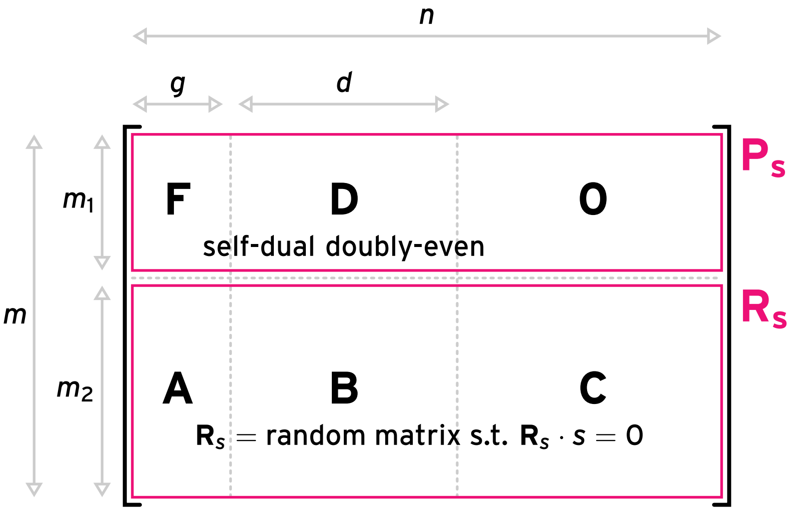

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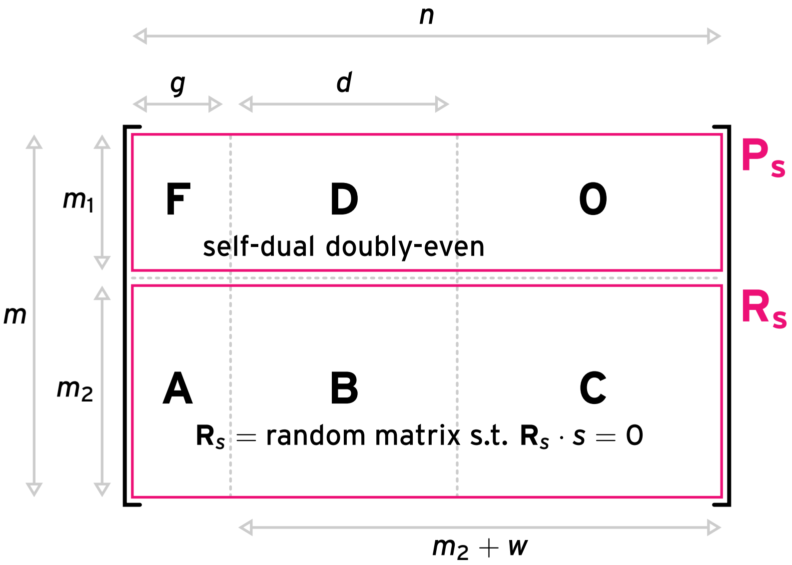

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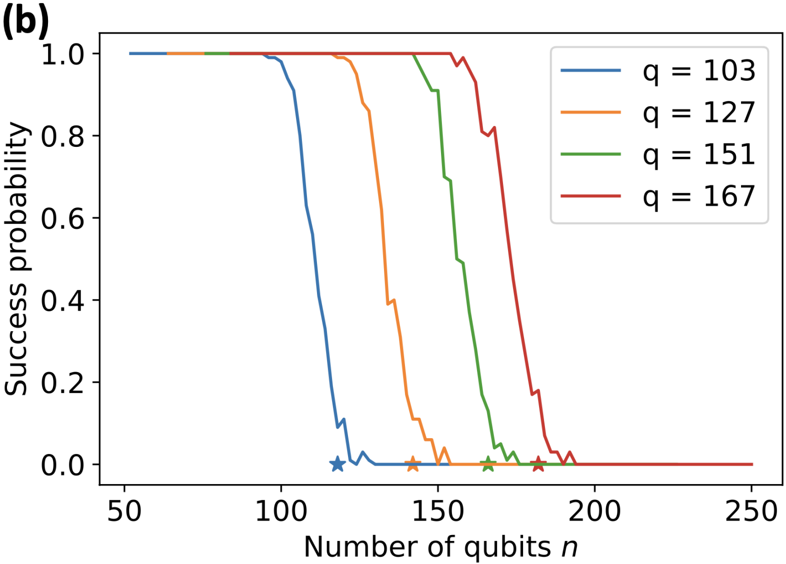

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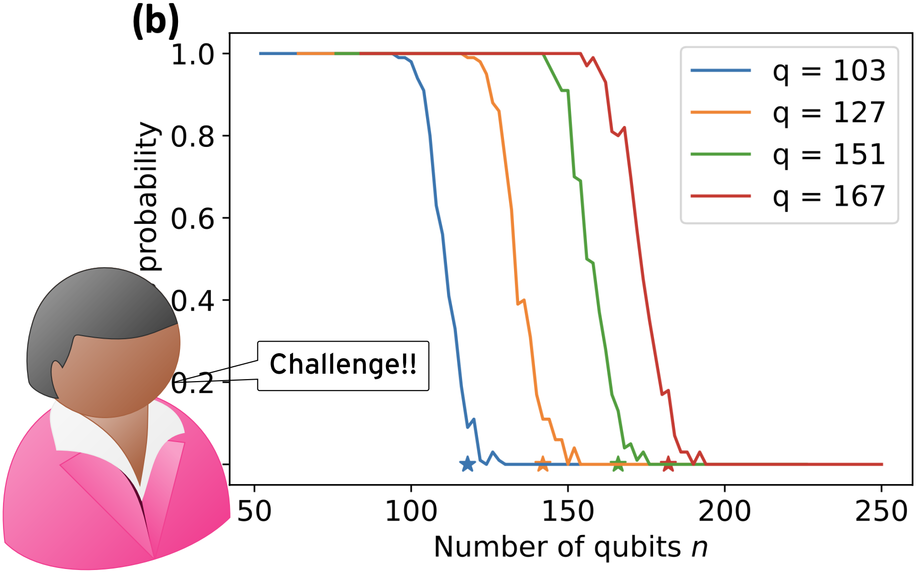

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ACT III

Hope for IQP is waning

The radical attack

$$\begin{bmatrix} \mathbf{F} & \mathbf{D} & \mathbf{0} \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{bmatrix}$$

- Range is unchanged under column operations.
- If range$[\mathbf{B}|\mathbf{C}] = \mathbb{F}_2^{m_2}$, can 'clear' columns below $[\mathbf{F}|\mathbf{D}]$.
- Elements of $\mathbf{D}'$ are in the radical of $\mathbf{H}$!
- The support of $\mathrm{col}(\mathbf{D}')$ determines the secret.

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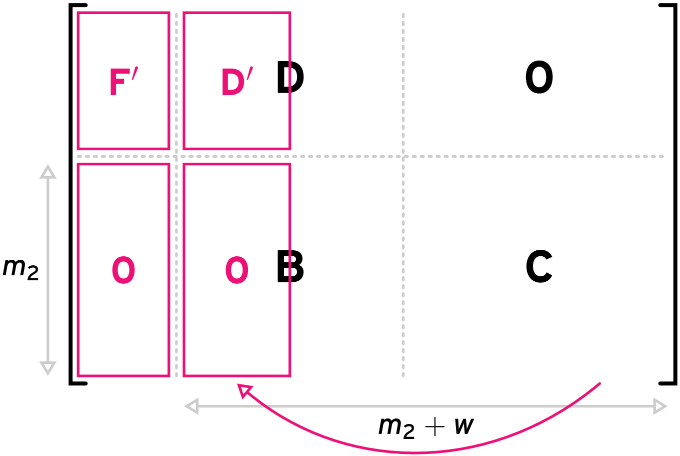

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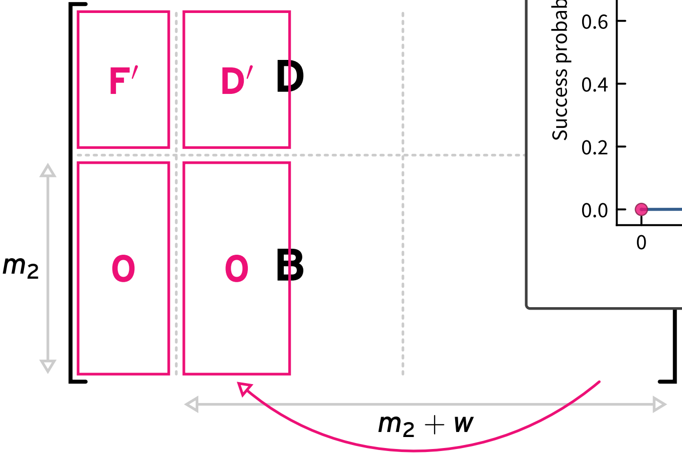

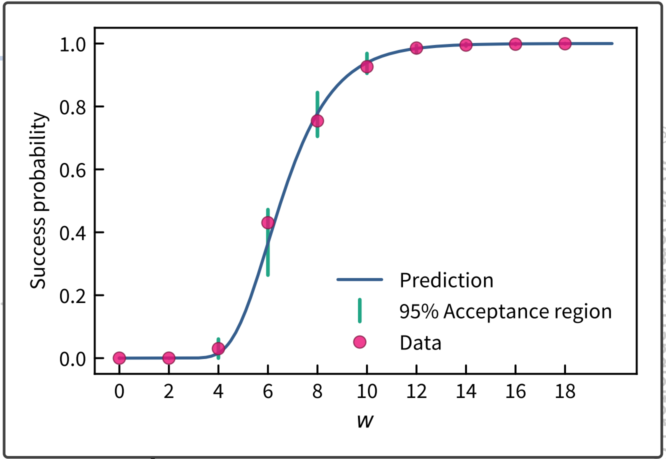

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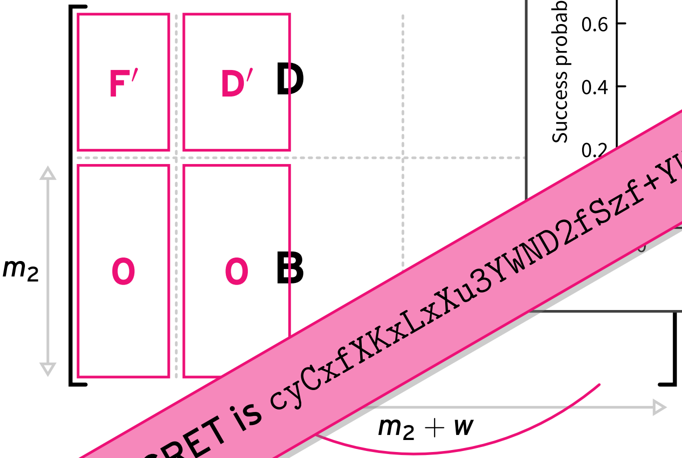
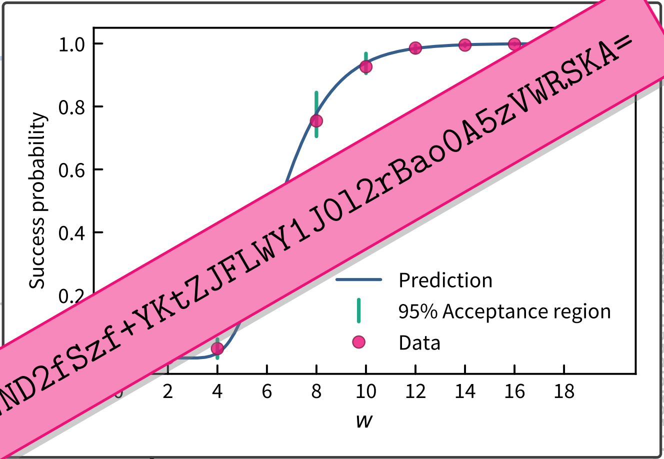
The radical attack





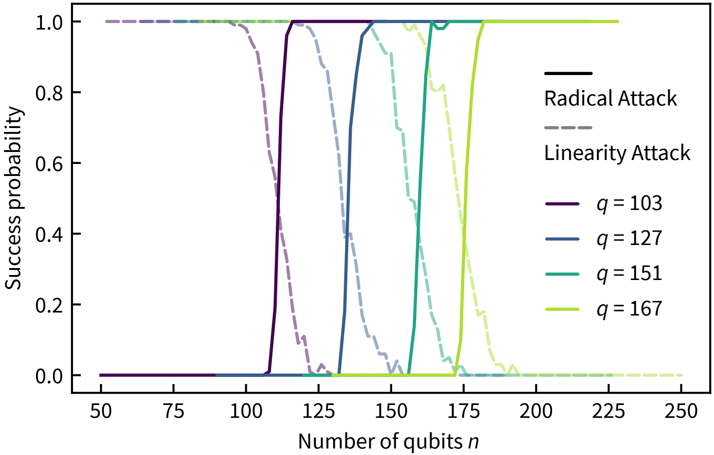
→ The support of col($\mathbf{D}'$) determines the secret.

The SECRET is cyCxfXKxLxXu3YWND2fSzf+YKtZJFLWY1J012rBao0A5zVWRSKA=

…023] Bremner, Cheng, Ji, arXiv:2308.07152.

More attacks

→ **Valleys of opportunity!**



More attacks

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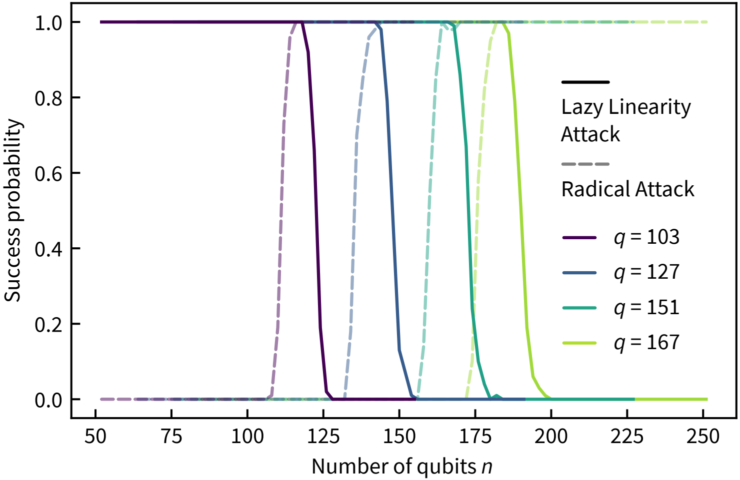

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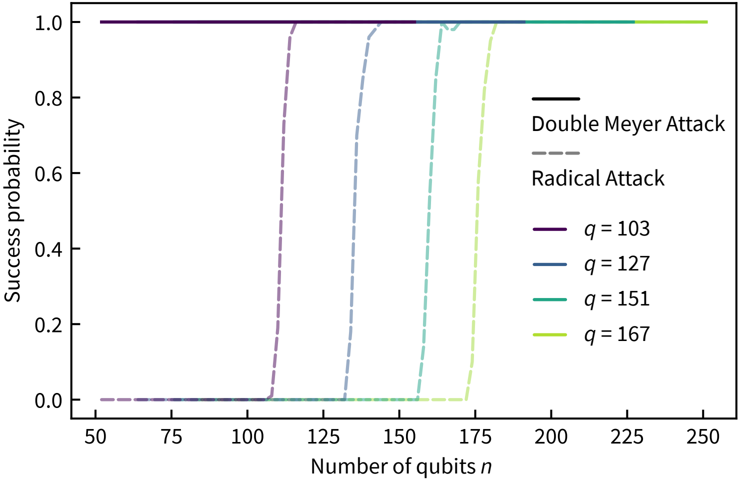

More attacks

- The **Lazy Meyer Attack:** Only search small kernels
- The **Double Meyer Attack:** Take kernel intersections to make the search space smaller
- **Hamming's razor:** identify redundant rows by exploiting that there are no low-weight Hamming strings in the image of the secret space.

THE END

Outlook

Hiding secrets

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- Nonlinear tests?
- Can we apply similar ideas to universal circuits?
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  [Aaronson-Zhang-24]

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Using interaction

- → Are there less structured interactive schemes?
- → E.g. mid-circuit measurements in a random circuit with a little bit of structure?